Dynamic ESG Equilibrium

Doron Avramov* Abraham Lioui† Yang Liu‡ Andrea Tarelli§

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Abstract

This paper develops and applies an equilibrium model that accounts for ESG demand and supply dynamics. In equilibrium, ESG preference shocks represent a novel risk source characterized by diminishing marginal utility and positive premium. Expected green asset returns are negatively associated with time-varying convenience yield, while exposures to ESG preference shocks lead to positive green premia. Augmenting these conflicting forces with positive contemporaneous effects of preference shocks on realized returns, the green-minus-brown portfolio delivers large positive payoffs for reasonably long horizons. Nonpecuniary benefits from ESG investing account for a nontrivial and increasing fraction of total consumption.

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*Interdisciplinary Center (IDC), Herzliya, Israel. doron.avramov@idc.ac.il
†EDHEC Business School, Nice, France. abraham.lioui@edhec.edu
‡The University of Hong Kong, Hong Kong, China. yangliu5@hku.hk
§Catholic University, Milan, Italy. andrea.tarelli@unicatt.it

1 Introduction

In 2020, sustainable funds pulled $370 billion in new money, more than twice the amount in 2019, while flows for the first half of 2021 are already at $320 billion. In addition, as of June 2021, the combined assets managed by sustainable funds reached $2.25 trillion, up from about $700 billion at the end of 2018.[1] While the exponential growth in sustainable investing has been quite consistent over recent years, the coronavirus pandemic has even intensified discussions about the interconnectedness of sustainability and capital markets. For one, J.P. Morgan argues in a July 2020 research letter that the pandemic and the destruction left in its wake could lead to a greater adoption of impact investing.[2] The notion is that some policymakers and investors perceive the crisis as a wake-up call that accelerates the quest for a different investment philosophy, as parallels have been drawn between the unforeseen risks of a pandemic and risks associated with climate change. Sustainable investors aim to generate measurable social and environmental impacts along with financial returns. Incremental value is attributed to the role that sustainable assets play in reshaping global standards.

The asset pricing literature has responded to the growing interest in social investing. Pástor et al. (2021a) consider an agent who derives nonpecuniary benefits from holding green stocks. They propose a CAPM representation for the cross section of average returns, with alpha that is inversely related to the ESG score of a firm, and they also motivate an alternative specification where the market portfolio is augmented by an ESG factor. Avramov et al. (2021) account for uncertainty about the correct ESG profile of a firm in analyzing investment decisions and asset pricing. Berk and van Binsbergen (2021) consider a market where a fraction of investors is ESG sensitive and study the impact of ESG divestitures on the cost of capital. Notably, all these studies formulate a single-period equilibrium.

This paper develops and implements an equilibrium model that accounts for ESG demand and supply dynamics. The model applies to both the aggregate market and the cross section of individual assets. A dynamic model is motivated on several grounds. In the first, it can naturally accommodate preference shocks for sustainable investing. Preference shocks reflect the unexpected component of the growing interest in sustainable investing over recent years. While preference shocks are advocated in a general context by Albuquerque et al. (2016) and Schorfheide et al. (2018), we focus on ESG-

[1] Figures are from Morningstar.

[2] https://www.jpmorgan.com/insights/research/covid-19-esg-investing.

related shocks, consistent with evidence. Second, a dynamic model can also account for supply shocks. The market ESG profile represents the supply side. More sustainable products and services as well as advanced technological breakthroughs (e.g., technological innovations for building sustainable cities, cars, and plants) are mapped into growing supply. Third, the nonpecuniary benefits from social investing could vary with the state of the economy. For instance, Bansal et al. (2021) document that preference for socially-responsible investing is procyclical. Finally, dynamic models have been more successful in capturing asset pricing regularities, such as the high equity premium, the low risk free rate, and the excess volatility (e.g., Weil, 1989; Bansal and Yaron, 2004). As we develop the theory section, we show that the proposed equilibrium provides incremental insights about the asset pricing implications of sustainable investing, while empirical experiments reinforce the lessons.

The model proceeds as follows. Agent's preferences are formulated through a modified version of Epstein and Zin (1989, 1991) that accounts for the ESG profile of the investment universe. In addition to deriving utility from the physical consumption, the agent extracts nonfinancial benefits from holding green assets through a two-good economy. ESG benefits establish the second good, while its share in the overall consumption bundle depends on the demand and the supply of sustainable investing and their time-series dynamics. The two-good economy extends the traditional consumption CAPM of Lucas (1978) and Breeden (1979). For instance, the second good is a luxury good in Ait-Sahalia et al. (2004), the service flow of durable goods in Yogo (2006), housing in Piazzesi et al. (2007), leisure in Van Binsbergen et al. (2012), and money in Bakshi and Chen (1996) and Lioui and Maio (2014).

There are three factors driving the risk premia in the economy. The first is consumption growth, as in standard consumption based models, while the others are ESG related. The second factor reflects the return on aggregate wealth, as in standard recursive preferences, but there is an important difference. In particular, when the agent is brown averse and the market is green, the effective (ESG-adjusted) return on wealth is perceived higher than the physical return. A higher preference for sustainable investing leads to the same outcome. The third factor represents the growth in the ratio of total consumption bundle to physical consumption and is related to the intertemporal variation of the aggregate benefits from ESG investing relative to physical consumption. The third factor characterizes two good economies. In our setup, because the consumption stream and the trading strategy that defines the second consumption good are nonseparable (which would also apply under time-additive preferences), the third factor evolves endogenously.

Beyond the incremental contribution of the two ESG related factors, there is also a convenience yield effect, which reflects the notion that a brown-averse agent is willing to compromise on a lower risk premium when holding green assets. The convenience yield terminology is adopted from Krishnamurthy and Vissing-Jorgensen (2012), who formulate consumption that contains convenience benefits from investing in liquid and safe U.S. Treasuries. In our setup, the convenience yield effect echoes the negative ESG-alpha relation in Pástor et al. (2021a).

The dynamic setup offers several insights. First, the convenience yield is not fixed but rather it varies with ESG demand and supply. Second, as the agent's value function is concave in both ESG demand and supply, positive demand or supply shocks are associated with diminishing marginal utility. We show that as the market gets more green, a brown-averse agent becomes more sensitive to ESG demand and supply shocks and would thus require a higher risk premium for holding the market. The required premium increases with the volatility of demand and supply shocks. Moreover, green assets are associated with a positive risk premium due to ESG demand shocks, while negative risk premium applies to brown assets. The risk premium channel, hence, challenges the negative ESG-expected return relation that characterizes the static setup. Taken together, the ESG-expected return relation fluctuates due to time variation in the convenience yield component, and can eventually go either way.

Next, we take the model to data. The sample spans the 1992 through 2020 period. The model can be represented through a linear state space obtained by stacking the dynamics of consumption growth, aggregate ESG supply, aggregate ESG demand, portfolio ESG scores and excess returns, and the market excess return. We consider green, brown, and green-neutral portfolios along with the market portfolio. The joint dynamics is described through structural vector autoregression of order one. Because some of the dynamics (e.g., ESG demand) are unobserved, we use the Kalman filter for estimating the model parameters. We first note that the market-implied estimate for the ESG preference displays time-series patterns that closely follow trends in the interest in ESG investing, as reflected by other sentiment measures based on the number of web searches through Google Trends or press attention in Factiva records.

In a dynamic model, there is a wedge between realized and expected returns. In particular, the model-implied average expected excess return of the green portfolio is 7.38%, while it is higher at 8.29% for the brown. The green-minus-brown portfolio has a negative and statistically significant average expected return of -0.91% per annum. However, throughout the sample, the unexpected shocks to ESG demand induce a positive unexpected return that adds to the conditional expected return of green assets. Considering the combined effect of the conditional expected return and the unexpected return due to demand shocks, the green-minus-brown portfolio average return is minor at -0.05% and insignificant, consistent with the negligible spread observed in the data.

Then, over recent years, the shift in tastes for ESG investing plays a meaningful role on the realized return of the green-minus-brown portfolio. To illustrate, between 2018 and 2020, the average conditional expected return of the green-minus-brown portfolio is negative at -0.97% per annum, while the model-implied annual return accounting also for ESG demand shocks is 8.39%, close to the realized value of 7.19%. As the impact of unanticipated ESG demand shocks on realized returns can be sizable, this calls for caution when inferring future returns of ESG investments based on past realized returns. If anything, due to increasing convenience yield, future returns of green assets are expected to diminish. By providing a structural description for the relation between unexpected shocks to ESG demand and realized returns, our work lends support to the findings in Pástor et al. (2021b), who highlight the positive association between shifts in environmental concerns and unexpected returns of environmentally-friendly stocks.

We further illustrate the expected-realized return gap. While the realized return of the green-minus-brown portfolio is around zero, during the entire sample, once we exclude the effect of ESG demand shocks on realized return, the cumulative return drops to -28%. In addition, from impulse response experiments based on the sample estimated parameters, we learn that the cumulative return of the green-minus-brown portfolio is at 6% following a positive one standard deviation annual preference shock. The positive effect of realized returns vanishes only after about six years following the end of the shock. Hence, with the positive contemporaneous effects of preference shocks on realized returns, the green-minus-brown portfolio could deliver large positive average returns over reasonably long horizons.

We finally show that the ESG benefits could be considerable from the perspective of a brown averse agent. In particular, throughout the entire sample, the estimated ESG benefits amount to 0.74% of total consumption, which is significant at conventional levels. Focusing on the most recent year, 2020, the benefits are already in 5.00% due to advancing levels of ESG demand, with an upward trend.

This paper contributes to several strands of the literature. First, we account for the dynamic nature of ESG demand and supply in equilibrium asset pricing for both the aggregate market and the cross section. Prior work has considered ESG preferences in single-period setups with static ESG preferences (e.g., Heinkel et al., 2001; Avramov et al., 2021) or with a possible transition of ESG tastes across generations (e.g., Pástor et al., 2021a,b), while our model accounts for ESG demand and supply dynamics. We show that demand (preference) shocks represent a novel risk source characterized by diminishing marginal utility and positive premium, while, empirically, supply shocks are only second order. In addition to providing a structural relation between ESG shocks and realized returns, our model highlights the conflicting forces that govern (i) the expected return spread on the green-minus-brown portfolio and (ii) the gap between expected and realized returns associated with impact investing.

We also contribute to the growing literature on the cross-sectional return predictability of the ESG profile. Prior studies show weak return predictability of the overall ESG rating (e.g., Pedersen et al., 2021) and mixed evidence based on different ESG proxies (e.g., Gompers et al., 2003; Hong and Kacperczyk, 2009; Edmans, 2011; Bolton and Kacperczyk, 2021). While Avramov et al. (2021) propose that ESG uncertainty could tilt the ESG-performance relation, this paper shows that ESG demand and supply shocks entail risk premia that could offset the negative ESG-expected return relation implied by the convenience yield of green assets. Augmenting these conflicting forces with positive contemporaneous effects of preference shocks on realized returns, the green-minus-brown portfolio could deliver large positive payoffs for reasonably long horizons. This paper is also the first to estimate the nonpecuniary benefits from ESG investing. We show that ESG benefits account for a nontrivial and increasing fraction of total consumption.

This work is also related to the literature studying asset pricing implications of the demand function for risky assets. Koijen and Yogo (2019) demonstrate that the cross section of stock returns is largely explained by latent demand shocks while only to a smaller extent by shocks related to changes in firm characteristics. Similarly, in our setup, ESG demand is a latent variable that plays a meaningful role in determining realized and expected returns of green and brown assets. In addition, future work can assess the demand implications of extending the set of characteristics to include variables associated with impact investing, such as ESG scores and their associated uncertainties.

The remainder of this paper is organized as follows. Section 2 presents the economic setting. Section 3 describes the data. Section 4 introduces the estimation technique, describes the parameter estimates and model-implied asset returns, and displays the time series implications of the model. Section 5 explores the quantitative implications of the estimated model, analyzing the impact of unexpected shocks to ESG supply and demand on asset prices and returns. The conclusion follows in Section 6.

2 Economic setting

This section develops the paradigm for ESG dynamic equilibrium. We formulate preferences that account for the notion that economic agents might benefit from investing in sustainable assets in ways that are not captured by the physical consumption stream. Based on the proposed preferences, we present and analyze general expressions for the stochastic discount factor and the risk premium. We then impose structure on equilibrium, including the dynamics of demand and supply for sustainable investing, to develop interpretable expressions for return on the wealth portfolio, the market premium, and the cross section of asset returns. Special attention is paid to understanding the realized and expected return spread between green and brown assets.

We first describe the trends in the recognition of ESG themes among investors, institutions, individuals, and corporations.

2.1 Trends in the recognition of sustainability

Figure 1a depicts the exponential growth in the number of Google searches for ESG and ESG investing terms, while Figure 1b shows the number of newspaper articles in the Dow Jones Factiva database that include keywords on sustainable investing, relative to the total number of articles containing keywords on investing.[3] The relative number of articles about sustainable investing has increased significantly from the beginning of the sample until 2002, then dropped following the dot-com bubble burst and the 2008 financial crisis, and has consistently advanced during the recent years. The pattern is also consistent with the procyclical nature of preference for socially-responsible investments documented in Bansal et al. (2021).

Likewise, Figure 1c suggests that new money flows into sustainable funds have considerably trended upward. Remarkably, in 2020, flows into U.S. sustainable funds reached $51.1 billion in new money, more than twice the amount of $21.4 billion in 2019. Sustainable funds account for a steadily increasing fraction of the assets under management of the U.S. mutual funds industry. Investors purchasing sustainable funds could be interested not only in the financial outcomes, e.g, the risk-return profile, of their portfolios, but also in the role that sustainable assets play in reshaping global standards, primarily global warming but also inadequate social impact and governance.

As we discuss later in the text, the aggregate market has become more green over

[3]A similar analysis, for the period 1982–2009, is conducted by Capelle-Blancard and Monjon (2012).

recent years. Profit maximizing corporations respond to ESG trends for various reasons including tax benefits, innovation stimulus, access to loans and grants, reduced cost of equity capital, and the attempt to cater to customers who prefer corporations with a strong enough environmental reputation. Focusing on the latter, the 2006 Cone Millennial Cause Study suggests that millennials are more likely to trust a company that has a reputation of being socially or environmentally responsible, while they are willing to turn down a product or service from an otherwise socially or environmentally irresponsible firm.[4]

2.2 Preferences

We consider an economy endowed with an infinitely-lived representative agent, who chooses a life-time consumption stream along with a trading strategy denoted by the N vector of portfolio weights $\boldsymbol{\omega}_t = [\omega_1, \omega_2, \ldots, \omega_N]'$, where N is the number of risky assets and t is a time subscript. There is also a risk-free asset in zero net supply. The agent's preferences are formulated through a modified version of Epstein and Zin (1989, 1991) that accounts for the ESG profile of the investment universe. In particular, in addition to deriving utility from the physical consumption, the agent extracts non-financial benefits from holding green assets. The amount of ESG benefits depends on the demand for sustainable investing, the supply of ESG investments, and the time-series dynamics of ESG demand and supply.

The agent solves the optimization problem

$$U_t = \max_{C_t, \boldsymbol{\omega}_t} \left((1-\beta) A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{1}$$

$$A_t = C_t + \delta_t G_{W,t} \left(W_t - C_t \right), \tag{2}$$

$$W_{t+1} = \left(W_t - C_t \right) \left(R_{f,t+1} + \sum_{n=1}^{N} \omega_{n,t} \left(R_{n,t+1} - R_{f,t+1} \right) \right), \tag{3}$$

where U_t stands for the value function, A_t is a consumption bundle that we describe below, C_t denotes the physical consumption, W_t is the aggregate wealth prior to consumption, $W_t - C_t$ is the investable wealth, $R_{n,t+1}$ is the gross return on the n-th risky security, $R_{f,t+1}$ is the risk-free gross return, $\mathrm{E}_t[.]$ stands for the conditional expectation operator, $G_{W,t} = \sum_{n=1}^{N} \omega_{n,t} G_{n,t}$ is the aggregate ESG supply, $G_{n,t}$ is the ESG score of the n-th

[4] http://www.conecomm.com/2006-cone-communications-millennial-cause-study-pdf.

asset, with positive (negative) values representing green (brown) assets. The zero case corresponds to ESG neutrality. The risk-free asset is assumed, without loss of generality, to be ESG neutral.[5]

Preference parameters are as follows. β is the subjective discount factor, ψ is the intertemporal elasticity of substitution, γ is a measure of relative risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and δ_t stands for ESG preferences, with a positive value representing brown aversion and with higher values representing stronger preferences. The parameter δ_t also quantifies the ESG share in the consumption bundle A_t. Innovations in δ_t represent preference shocks for sustainable investing.

ESG considerations are characterized by demand and supply forces. The brown aversion parameter, δ_t, captures the demand for sustainable investing. The demand increases with the growing concerns on global warming, inadequate governance, or social inequality. The ESG score of aggregate wealth $G_{W,t}$ represents the supply side. More sustainable products and services as well as advanced technological breakthroughs are mapped into higher supply. Both the demand and the supply are time varying and summarize the evolutionary nature of ESG in the structural interpretation.

The consumption bundle A_t replaces the consumption good C_t in the original specification of Epstein and Zin (1989, 1991). The bundle consists of the physical good C_t and an incremental consumption good that evolves from nonpecuniary benefits associated with sustainable investing. The ESG-based good is equal to the product of the brown aversion δ_t, the greenness of aggregate wealth $G_{W,t}$, and the total amount of invested wealth $W_t - C_t$. From the perspective of a brown-averse agent, a positive aggregate greenness $G_{W,t}$ makes the consumption bundle A_t more valuable than the physical consumption C_t. The opposite holds for negative aggregate greenness. ESG externalities are proportional to the amount of wealth invested in all financial assets, including the wealth portfolio that represents a claim on the consumption stream.

The specification for the consumption bundle A_t is a particular case of a common setup accounting for two goods with a constant elasticity of substitution, ρ, and the share of the second good that is equal to δ: $A_t = (C_t^{1-\frac{1}{\rho}} + \delta \left(G_{W,t}\left(W_t - C_t\right)\right)^{1-\frac{1}{\rho}})^{\frac{1}{1-\frac{1}{\rho}}}$. Relative to the existing literature, we consider a limiting case of an infinite elasticity of substitution, which results in the linear expression in equation (2), while we introduce the flexibility that δ_t could be time varying.

In the setup developed here, consumption and portfolio choice are nonseparable, as

[5] The equilibrium results hold also when the risk-free asset has some color while, then, $G_{n,t}$ stands for the asset ESG score in excess of the risk-free ESG.

portfolio weights affect the value of ESG-based consumption in the overall bundle. Thus, the incremental asset pricing effects of sustainable investing evolve endogenously.

2.3 Dynamic ESG equilibrium: A general outlook

We first derive general asset pricing outcomes based on equations (1) through (3). In particular, we solve for the stochastic discount factor (SDF) as well as the Euler equation and the risk premium for a generic asset, be it the aggregate wealth portfolio, the market portfolio, or any individual asset. All these quantities are described in Proposition 1, while Section A of the Online Appendix provides technical details for the derivation.

Proposition 1. *In equilibrium, the Euler equation for the gross return on a generic asset n with an ESG score equal to G_n is given by*

$$\mathrm{E}_t\left[M_{t+1}R_{n,t+1}\right] = 1 - \delta_t G_{n,t}, \tag{4}$$

where M_{t+1}, the SDF, is formulated as

$$M_{t+1} = \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}} \tilde{R}_{W,t+1}^{\theta-1}\left(\frac{1+\delta_{t+1}G_{W,t+1}\frac{W_{t+1}-C_{t+1}}{C_{t+1}}}{1+\delta_t G_{W,t}\frac{W_t-C_t}{C_t}}\right)^{-\frac{\theta}{\psi}}, \tag{5}$$

and $\tilde{R}_{W,t+1} = \frac{R_{W,t+1}}{1-\delta_t G_{W,t}}$ is the ESG-adjusted gross return on the consumption asset.[6] *The expected excess return on a generic asset n is given by*

$$\begin{aligned}
&\mathrm{E}_t\left[r_{n,t+1} - r_{f,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[r_{n,t+1}\right] = \\
&\quad \frac{\theta}{\psi}\mathrm{Cov}_t\left[\Delta c_{t+1}, r_{n,t+1}\right] + (1-\theta)\,\mathrm{Cov}_t\left[\tilde{r}_{W,t+1}, r_{n,t+1}\right] \\
&\quad + \frac{\theta}{\psi}\mathrm{Cov}_t\left[\log\left(\frac{1+\delta_{t+1}G_{W,t+1}\frac{W_{t+1}-C_{t+1}}{C_{t+1}}}{1+\delta_t G_{W,t}\frac{W_t-C_t}{C_t}}\right), r_{n,t+1}\right] - y_{n,t},
\end{aligned} \tag{6}$$

[6]The Euler equation (4) can be also written as

$$\mathrm{E}_t\left[\beta^{\theta}\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}} \tilde{R}_{W,t+1}^{\theta-1}\tilde{R}_{n,t+1}\right] = 1,$$

where $\tilde{R}_{n,t+1} = \frac{R_{n,t+1}}{1-\delta_t G_{n,t}}$ is the ESG-adjusted gross return on asset n. This expression resembles the traditional Epstein-Zin solution, where the total consumption bundle growth $\frac{A_{t+1}}{A_t}$ replaces the consumption growth $\frac{C_{t+1}}{C_t}$, and ESG-adjusted gross returns $\tilde{R}_{W,t+1}$ and $\tilde{R}_{n,t+1}$ replace gross returns $R_{W,t+1}$ and $R_{n,t+1}$.

where $\text{Cov}_t[.,.]$ *denotes the time-t conditional covariance,* $\Delta c_{t+1} = \log \frac{C_{t+1}}{C_t}$*,* $r_{n,t+1} = \log R_{n,t+1}$*,* $r_{f,t+1} = \log R_{f,t+1}$*,* $\tilde{r}_{W,t+1} = \log \tilde{R}_{W,t+1}$*, and* $y_{n,t} = -\log(1 - \delta_t G_{n,t})$*.*

The closed-form solutions for the SDF and expected excess return reinforce the Epstein-Zin tractability for asset pricing even in the presence of an incremental source of nonseparability, namely, between the physical consumption and the trading strategy.[7] There are three factors driving the risk premia in the economy. While the first, the consumption growth of the physical goods, $\log \frac{C_{t+1}}{C_t}$, is standard in consumption based models, the other two already account for ESG dynamics. In particular, the second factor, which is exclusively attributed to time nonadditive preferences, suggests that the return on aggregate wealth is ESG adjusted by $\tilde{R}_{W,t+1} = \frac{R_{W,t+1}}{1-\delta_t G_{W,t}}$. From the perspective of a brown-averse agent, a positive aggregate greenness $G_{W,t}$ makes the perceived return on aggregate wealth, $\tilde{R}_{W,t+1}$, higher than the actual return on wealth, $R_{W,t+1}$. A higher preference for sustainable investing (higher δ_t) leads to the same outcome. The third factor represents the growth in the ratio of total consumption bundle to physical consumption, $\frac{A_{t+1}/C_{t+1}}{A_t/C_t} = \frac{1+\delta_{t+1}G_{W,t+1}(W_{t+1}-C_{t+1})/C_{t+1}}{1+\delta_t G_{W,t}(W_t - C_t)/C_t}$. It is thus related to the intertemporal variation of the aggregate benefits from ESG investing relative to physical consumption, and depends on the variation of both demand for ESG through δ_t and aggregate supply through $G_{W,t}$.

We next analyze ESG implications for expected asset returns. For a green-neutral asset, the right hand side of (4) equals one, as in standard setups. Otherwise, the right-hand-side is lower than one when the asset is green and higher than one when it is brown. In the same vein, observe from equation (6) that a green (brown) asset carries a positive (negative) convenience yield $y_{n,t}$, which is increasing in $\delta_t G_{n,t}$. The convenience yield reflects the interaction between agent's ESG preferences and the asset ESG profile. The presence of convenience yield suggests that a brown-averse agent is willing to compromise on the risk premium of a green asset due to nonmonetary benefits from holding the asset. Thus, green assets have positive convenience yield which translates into lower expected return.

The convenience yield effect echoes the negative ESG-alpha relation in Pástor et al. (2021a).[8] However, there are important differences in the dynamic setup. First, the

[7]While there is no time separability in Epstein-Zin preferences, there is separability between the consumption and portfolio choice. See equation 5.6 on page 955 of Epstein and Zin (1989) and the discussion that follows. In our setup, utility of consumption and portfolio choice are nonseparable because the portfolio choice defines the current utility. This additional source of nonseparability would also emerge under time-additive preferences.

[8]Heinkel et al. (2001) also develop a one-period model where polluting firms have a higher cost of capital. Hong and Kacperczyk (2009) provide evidence that "sin" stocks have higher expected returns

convenience yield is not fixed but rather it varies with the ESG demand and supply. Second, beyond convenience yield, the expected return is based on covariances of returns with two ESG related asset pricing factors. The risk premium channel can either reinforce or challenge the negative ESG-expected return relation.

To analyze risk premium implications, we first note that the value function (1) is concave in both ESG demand and supply.[9] Thus, positive shocks to δ_t or $G_{W,t}$ are associated with diminishing marginal utility, suggesting that the incremental ESG benefits from holding green assets drop when the market gets greener. As ESG demand or supply shocks are negatively correlated with the SDF, assets with returns that are positively correlated with ESG demand or supply shocks deliver a positive ESG-induced risk premium. The opposite applies to negatively correlated assets. To further analyze the risk premium, we consider the case where $\psi > 1$ ($\theta < 0$) which implies preference for early resolution of uncertainty and is consistent with a large body of work, including Bansal and Yaron (2004), Albuquerque et al. (2016), and Schorfheide et al. (2018).

The positive risk premium due to ESG shocks evolves from the combined effect of the second and third factors in (5). The correlation between ESG shocks and the third factor is positive, implying a negative risk premium due to ESG shocks.[10] The second factor is negatively correlated with the ESG-adjusted return on aggregate wealth, $\tilde{R}_{W,t+1}$. Shocks to ESG demand or supply have a contemporaneous effect on the return on aggregate wealth, and thus on the SDF and risk premium, through the second factor. The direction and magnitude of that contemporaneous effect on the risk premium cannot be inferred directly. However, because ESG shocks carry a positive risk premium due to the concavity of preferences, two indirect inferences can be made. First, the second factor must be negatively correlated with the stochastic discount factor and is thus associated with a positive risk premium. Second, the risk premium due to the second factor must be greater, in absolute value, than the negative contribution of the third factor. As we later formulate standard dynamics for consumption growth, ESG preference parameter, and ESG scores, we confirm that the return on aggregate wealth is indeed positively correlated with ESG demand and supply shocks.

It should be noted that the positive risk premium due to ESG demand and supply shocks is not limited to the case of time nonseparable preferences. To illustrate, consider

than stocks with otherwise similar characteristics, while Bolton and Kacperczyk (2021) find evidence of a carbon premium which is paid by firms characterized by higher CO2 emissions.

[9]See derivation in Online Appendix A.

[10]From the model estimation, we confirm that the indirect dependence of the third factor on the price-to-consumption ratio is second order relative to the direct and positive impact of ESG shocks.

the time additive case, where $\theta = 1$. Then, the second factor vanishes, while the exponent of the third factor is $-\gamma$. Hence, the SDF is negatively correlated with ESG supply and demand shocks, amounting to a positive risk premium.

Taken together, assets realizing returns that are positively related to ESG shocks are characterized by a positive risk premium component, while the opposite applies to negative exposure assets. At this general stage, the ESG-expected return relation is inconclusive, in the absence of information about the precise exposures of green and brown assets to ESG shocks. Nevertheless, by imposing reasonable structure on the economy (in the subsection that follows), we are able to qualify the ESG-induced risk premium and highlight its direction and determinants.

In particular, we show that, as the market gets greener, a brown-averse agent becomes more sensitive to ESG demand and supply shocks and would thus require a higher risk premium for holding the market. The required premium increases with the volatility of demand and supply shocks. Moreover, green assets are associated with a positive risk premium due to ESG demand shocks, while negative risk premium applies to brown assets. The risk premium channel, hence, challenges the negative ESG-expected return relation that characterizes the static setup. We provide details below.

2.4 Imposing structure on equilibrium

We formulate exogenous processes for consumption growth Δc_t, the aggregate greenness $G_{W,t}$, and the ESG preference parameter δ_t. For ease of interpretation, we retain the assumption that $\psi > 1$ and further assume that $\bar{\delta} > 0$ and $\bar{G}_W \geq 0$. That is, in a steady-state equilibrium, the agent is brown averse and the wealth portfolio is green (or green neutral). The exogenous processes are given by

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_c \varepsilon_{c,t+1}, \tag{7}$$

$$x_{t+1} = \rho_x x_t + \sigma_x \varepsilon_{x,t+1}, \tag{8}$$

$$G_{W,t+1} = \mu_G + \rho_G G_{W,t} + \sigma_G \varepsilon_{G,t+1}, \tag{9}$$

$$\delta_{t+1} = \mu_\delta + \rho_\delta \delta_t + \sigma_\delta \varepsilon_{\delta,t+1}. \tag{10}$$

The consumption growth process (7) is homoskedastic.[11] As in Bansal and Yaron

[11]While accounting for stochastic volatility would introduce an additional source of time variation in the equity premium, we focus on the incremental implications of ESG demand and supply, both of which do not interact with the volatility process. Individual stocks could still have heterogeneous loadings on volatility risk, while such loadings are independent of ESG characteristics. For parsimony, we thus

(2004), consumption growth has a predictable component that is driven by the mean-reverting long-run risk variable x_t (assuming $0 < \rho_x < 1$). Note that, while a persistent long-run risk variable x_t is useful to explain the constant component of the market premium, it does not interact with the ESG demand or supply. Therefore, our results with respect to the incremental asset pricing implications of sustainable investing are unchallenged when the long-run risk component is muted, i.e., when $\sigma_x = \rho_x = 0$, which amounts to identically and independently distributed (henceforth, IID) consumption growth. The processes (9) and (10) are also mean reverting (assuming $0 < \rho_G, \rho_\delta < 1$) with long-run means given by $\bar{G}_W = \frac{\mu_G}{1-\rho_G}$ and $\bar{\delta} = \frac{\mu_\delta}{1-\rho_\delta}$.[12] The innovations $\varepsilon_{c,t+1}$, $\varepsilon_{x,t+1}$, $\varepsilon_{G,t+1}$, and $\varepsilon_{\delta,t+1}$ are assumed to be IID normal with zero mean and unit variance, and uncorrelated with each other contemporaneously and in all leads and lags.

To derive equilibrium outcomes, we first log-linearize the return on aggregate wealth

$$r_{W,t+1} \simeq \kappa_{rW,0} + \kappa_{rW,pc} pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{11}$$

where $pc_t = \log \frac{W_t - C_t}{C_t}$ is the logarithm of the price-to-consumption ratio (investable wealth relative to consumption), $\kappa_{rW,pc} = \frac{e^{\overline{pc}}}{1+e^{\overline{pc}}}$, and $\kappa_{rW,0} = \log\left(1 + e^{\overline{pc}}\right) - \kappa_{rW,pc}\overline{pc}$.

The following proposition describes the price-to-consumption ratio and characterizes the return on aggregate wealth and the SDF dynamics.

Proposition 2. *The equilibrium price-to-consumption ratio, the return on wealth, and the SDF dynamics are given by*

$$pc_t = A_{pc,0} + A_{pc,G} G_{W,t} + A_{pc,\delta}\delta_t + A_{pc,x} x_t, \tag{12}$$

$$\begin{aligned} r_{W,t+1} = {} & r_{W,0} - A_{pc,G}\left(1 - \kappa_{rW,pc}\rho_G\right) G_{W,t} - A_{pc,\delta}\left(1 - \kappa_{rW,pc}\rho_\delta\right)\delta_t \\ & + \left(1 - A_{pc,x}\left(1 - \kappa_{rW,pc}\rho_x\right)\right) x_t \\ & + A_{pc,G}\kappa_{rW,pc}\sigma_G\varepsilon_{G,t+1} + A_{pc,\delta}\kappa_{rW,pc}\sigma_\delta\varepsilon_{\delta,t+1} \\ & + A_{pc,x}\kappa_{rW,pc}\sigma_x\varepsilon_{x,t+1} + \sigma_c\varepsilon_{c,t+1}, \end{aligned} \tag{13}$$

$$\begin{aligned} m_{t+1} = {} & m_0 + m_G G_{W,t} + m_\delta\delta_t + m_x x_t \\ & - \lambda_c\varepsilon_{c,t+1} - \lambda_G\varepsilon_{G,t+1} - \lambda_\delta\varepsilon_{\delta,t+1} - \lambda_x\varepsilon_{x,t+1}, \end{aligned} \tag{14}$$

where all constant coefficients are described in Online Appendix B.

consider a homoskedastic framework.

[12] In the presence of mean-reverting state variables, there is a steady state equilibrium, which can be found through a fixed-point problem. Log-linearization is implemented to the steady state values. The steady state also guarantees that the solution does not explode due to nonstationarity.

The price-to-consumption ratio, the expected return of the wealth portfolio, and the SDF drift are all affine functions of the state variables, $G_{W,t}$, δ_t, and x_t. We show in Online Appendix B that the coefficients $A_{pc,G}$, $A_{pc,\delta}$, and $A_{pc,x}$ are all positive.

The positive coefficients have several implications. First, the price-to-consumption ratio positively covaries with the aggregate ESG supply $G_{W,t}$, ESG demand δ_t, and the long-run risk variable, x_t. Thus, for a given level of physical consumption, as the aggregate ESG supply or demand rises, the wealth portfolio becomes more valuable. Moreover, because $\kappa_{rW,pc}, \rho_G, \rho_\delta < 1$, the conditional expected return on wealth in (13) is negatively correlated with the current levels of $G_{W,t}$ and δ_t, while the unexpected component of $r_{W,t+1}$ is positively correlated with contemporaneous shocks to ESG demand and supply.

The mechanism suggests that an unexpected increase in aggregate ESG benefits is associated with a positive price pressure on the wealth portfolio and a higher contemporaneous realized return as well as a lower expected future return. The positive correlation between ESG demand/supply shocks and realized returns on the wealth portfolio confirms that the second factor in the SDF in (5) is negatively exposed to ESG shocks, and thus commands a positive risk premium.

Notice that the log-linearized dynamics of the SDF in (14) are driven by four sources of risk, namely, shocks to (i) short-run consumption growth, $\varepsilon_{c,t+1}$, (ii) ESG supply, $\varepsilon_{G,t+1}$, (iii) ESG demand, $\varepsilon_{\delta,t+1}$, and (iv) long-run consumption growth, $\varepsilon_{x,t+1}$. Revisiting the three factors driving the general form of the SDF in (5), the first factor is exposed to short-run consumption growth shocks only, while the two other factors are exposed to all four systematic risks through their dependence on the return on aggregate wealth. The market prices of the four risk sources, λ_c, λ_G, λ_δ, and λ_x, are constant and positive.[13] Thus, assets whose returns are positively (negatively) correlated with the systematic shocks deliver a positive (negative) risk premium.

We also show in Online Appendix B that $-1 < m_x < 0$ and $m_G, m_\delta > 0$. As these coefficients appear in the SDF dynamics, they also drive the equilibrium risk-free rate of return. The risk-free rate is described in the following proposition, while Online Appendix C provides technical details.

Proposition 3. *The risk-free rate of return between time t and time $t+1$ is an affine*

[13] The proof that λ_c and λ_x are positive is in the Online Appendix B. It is more challenging to directly prove that λ_G and λ_δ are positive. Still, as discussed in Section 2.3, these risk premia must be positive due to the concavity of the value function with respect to variations of $G_{W,t}$ and δ_t. Furthermore, we verify that λ_G and λ_δ are positive in the context of the log-linearized model for a wide range of parameter values. The estimation also provides supporting evidence.

function of the state variables, $G_{W,t}$, δ_t, and x_t

$$r_{f,t+1} = -m_0 - \frac{\lambda_c^2}{2} - \frac{\lambda_G^2}{2} - \frac{\lambda_\delta^2}{2} - \frac{\lambda_x^2}{2} - m_G G_{W,t} - m_\delta \delta_t - m_x x_t. \tag{15}$$

As m_G and m_δ are positive, the risk-free rate is inversely related to the aggregate greenness and the ESG preference parameter. This is because, when the ESG demand or supply rises, the ESG-adjusted return of the wealth portfolio is perceived higher, while the expected financial returns of all assets, including the risk-free asset, diminish. The constant terms $-\frac{\lambda_G^2}{2}$ and $-\frac{\lambda_\delta^2}{2}$ follow because ESG supply and demand shocks contribute to the risk perceived by the agent. These shocks lead to higher precautionary saving motives, and are hence associated with lower risk-free interest rate. As $m_x < 0$, the risk-free rate is increasing in expected consumption growth, as higher financial benefits from investing in the wealth portfolio imply higher funding costs.

2.5 The market premium

To characterize the rate of return on the market portfolio, we assume that the market greenness is equal to that of aggregate wealth, i.e. $G_{M,t} = G_{W,t}$.[14] Then, the Euler condition (4) for the market portfolio is $\mathrm{E}_t\left[M_{t+1}R_{M,t+1}\right] = 1 - \delta_t G_{W,t}$.

Denoting by $pd_{M,t}$ the market price-to-dividend ratio, we implement the standard log-linearization for the market rate of return, which is given by

$$r_{M,t+1} \simeq \kappa_{rM,0} + \kappa_{rM,pd} pd_{M,t+1} - pd_{M,t} + \Delta d_{M,t+1}, \tag{16}$$

where $\kappa_{rM,pd} = \frac{e^{\overline{pd}_M}}{1+e^{\overline{pd}_M}}$ and $\kappa_{rM,0} = \log\left(1 + e^{\overline{pd}_M}\right) - \kappa_{rM,pd}\overline{pd}_M$. We next formulate the growth of the market logarithmic dividend process as

$$\Delta d_{M,t+1} = \mu_{dM} + \rho_{dM,x} x_t + \sigma_{dM,c}\varepsilon_{c,t+1} + \sigma_{dM}\varepsilon_{dM,t+1}, \tag{17}$$

where $\varepsilon_{dM,t+1}$ is IID normal and uncorrelated with the other innovations contemporaneously and in all leads and lags. As in Bansal and Yaron (2004), through the coefficient $\rho_{dM,x}$, the expected dividend growth is driven by a predictable component represented by the state variable that also drives consumption growth. We allow the unexpected component of dividend growth to vary with innovations in consumption growth, consistent

[14] We make this assumption for empirical tractability, as we rely on ESG scores of traded firms to assess the market ESG profile. The ESG profile of the wealth portfolio is unobservable. In the absence of this assumption, the market portfolio could be treated as any other risky asset.

with the empirical evidence in Xu (2021).

The following proposition characterizes the price-to-dividend ratio and the return of the market portfolio. The proof is in Online Appendix D. For ease of interpretation, the market portfolio is parsimoniously formulated, while the proof describes a more general case where the dividend process in (17) has a drift driven by δ_t and is allowed to be correlated with innovations in long-run risk and in aggregate ESG demand and supply.

Proposition 4. *The equilibrium price-to-dividend ratio of the market portfolio and the dynamics of the market return are given by*

$$pd_{M,t} = A_{M,0} + A_{M,G}G_{W,t} + A_{M,\delta}\delta_t + A_{M,x}x_t, \tag{18}$$

$$\begin{aligned} r_{M,t+1} &= r_{M,0} + (\kappa_{W,G} - m_G) G_{W,t} + (\kappa_{W,\delta} - m_\delta) \delta_t - m_x x_t \\ &\quad + \kappa_{rM,pd} A_{M,G} \sigma_G \varepsilon_{G,t+1} + \kappa_{rM,pd} A_{M,\delta} \sigma_\delta \varepsilon_{\delta,t+1} \\ &\quad + \kappa_{rM,pd} A_{M,x} \sigma_x \varepsilon_{x,t+1} + \sigma_{dM,c}\varepsilon_{c,t+1} + \sigma_{dM}\varepsilon_{dM,t+1}, \end{aligned} \tag{19}$$

where $A_{M,G} = \frac{m_G - \kappa_{W,G}}{1-\kappa_{rM,pd}\rho_G}$, $A_{M,\delta} = \frac{m_\delta - \kappa_{W,\delta}}{1-\kappa_{rM,pd}\rho_\delta}$, $A_{M,x} = \frac{m_x + \rho_{dM,x}}{1-\kappa_{rM,pd}\rho_x}$, $\kappa_{W,G} = -\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_W}$, *and* $\kappa_{W,\delta} = -\frac{\bar{G}_W}{1-\bar{\delta}\bar{G}_W}$. $A_{M,0}$ *and* $r_{M,0}$ *are formulated in Online Appendix D.*

The market price-to-dividend ratio and the market premium are also affine functions of the state variables $G_{W,t}$, δ_t, and x_t. Then, we show in Online Appendix D that both $A_{M,G}$ and $A_{M,\delta}$ are positive with $A_{M,G}$ increasing with the average brown aversion $\bar{\delta}$ and $A_{M,\delta}$ increasing with the average aggregate greenness $\bar{G}_W$. Hence, the market price-to-dividend ratio positively covaries with ESG demand and supply, suggesting that innovations in δ_t or $G_{W,t}$ have a positive contemporaneous effect on the price-to-dividend ratio. The sensitivity to long-run risk shocks, captured by $A_{M,x}$, is subject to two conflicting forces. First, a higher expected consumption growth implies a negative price effect due to stronger discounting of future cashflows. Then, there is a positive price effect due to a higher expected dividend growth. As suggested by prior work, the latter effect is typically stronger, hence, $A_{M,x}$ tends to be positive.

We turn to analyze the market premium. First, as $A_{M,G}$ and $A_{M,\delta}$ are positive, the realized market returns are positively correlated with contemporaneous shocks to ESG supply and demand. Then, taking conditional expectations of the realized return in equation (19) and subtracting the risk-free return in equation (15), the conditional market premium is given by

$$\mathrm{E}_t\left[\hat{r}_{M,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[\hat{r}_{M,t+1}\right] =$$

$$
\begin{aligned}
&\underbrace{\sigma_{dM,c}}_{\mathrm{Cov}_t\left[r_{M,t+1},\varepsilon_{c,t+1}\right]} \lambda_c + \underbrace{\kappa_{rM,pd} A_{M,G} \sigma_G}_{\mathrm{Cov}_t\left[r_{M,t+1},\varepsilon_{G,t+1}\right]} \lambda_G \\
&+ \underbrace{\kappa_{rM,pd} A_{M,\delta} \sigma_\delta}_{\mathrm{Cov}_t\left[r_{M,t+1},\varepsilon_{\delta,t+1}\right]} \lambda_\delta + \underbrace{\kappa_{rM,pd} A_{M,x} \sigma_x}_{\mathrm{Cov}_t\left[r_{M,t+1},\varepsilon_{x,t+1}\right]} \lambda_x \\
&+ \underbrace{\log\left(1 - \bar{\delta}\bar{G}_W\right) - \frac{\bar{\delta}\left(G_{W,t} - \bar{G}_W\right)}{1 - \bar{\delta}\bar{G}_W} - \frac{\bar{G}_W\left(\delta_t - \bar{\delta}\right)}{1 - \bar{\delta}\bar{G}_W}}_{-y_{M,t}}, \qquad (20)
\end{aligned}
$$

where $\hat{r}_{M,t+1} = r_{M,t+1} - r_{f,t+1}$ is the market excess return, $y_{M,t}$ is the convenience yield, the covariances stand for exposures to systematic risk factors, and lambdas describe the corresponding prices of risk. Risk exposures and risk premia are all positive and constant through time. Thus, the market premium consists of fixed risk premium and time-varying convenience yield. At the steady-state equilibrium (corresponding to $\delta_t = \bar{\delta} > 0$ and $G_{W,t} = \bar{G}_W \geq 0$), the convenience yield equals $-\log\left(1 - \bar{\delta}\bar{G}_W\right)$, which is positive. Hence, narrowing down the focus to the convenience yield premium, i.e., $-y_{M,t}$, the steady-state market premium is inversely associated with the ESG score. Moreover, through the time-varying component of the convenience yield, the conditional market premium diminishes with increasing ESG demand, δ_t, and supply, $G_{W,t}$.

Turning to the risk-based components, the risk premium associated with ESG supply shocks is represented by the second term in equation (20). As $A_{M,G}$ increases in $\bar{G}_W$, the risk premium rises with the market ESG profile. The notion is that a brown averse agent attributes more value to the market, when the market is greener. Hence, the agent is more exposed to ESG supply shocks, as reflected through stronger covariation between market return and ESG supply. Similarly, the third term in (20) represents the risk premium due to ESG demand shocks. As the price sensitivity to demand shocks, $A_{M,\delta}$, increases in $\bar{G}_W$, this further reinforces the positive ESG-risk premium relation. Moreover, the positive ESG-risk premium relation becomes stronger with an increasing volatility of ESG supply or demand shocks. It should be noted that the risk premium components associated with consumption and long-run risk shocks do not interact with the ESG risk premia. Thus, when consumption growth is IID, the term involving λ_x vanishes, while all other results remain intact.

In sum, by imposing a plausible structure on the economic processes for consumption growth, the market dividend growth, and the market ESG profile, we show that the risk premium component is positively associated with the ESG profile. Thus, the dynamic setup challenges the negative ESG-expected return relation that characterizes the single-period setup. From a comparative statics perspective, while the agent extracts benefits from holding green assets and is thus willing to compromise on a lower market premium (the convenience yield effect), the agent is also more sensitive to demand and supply ESG shocks if the market is greener, on average. With a high enough volatility of either ESG demand or ESG supply shocks, the positive effect on the ESG-expected return relation due to risk premium can dominate the negative effect due to convenience yield.

2.6 The return spread between green and brown assets

We next study the cross section of average returns and especially analyze the return spread between green and brown stocks. First, the dynamics of asset-specific greenness are formulated as

$$G_{n,t+1} = \mu_{Gn} + \rho_{Gn} G_{n,t} + \sigma_{Gn,G}\varepsilon_{G,t+1} + \sigma_{Gn}\varepsilon_{Gn,t+1}, \tag{21}$$

where $\varepsilon_{Gn,t+1}$ is IID normal and uncorrelated with all other innovations. When $0 < \rho_{Gn} < 1$, the process is mean-reverting with a long-run mean equal to $\bar{G}_n = \frac{\mu_{Gn}}{1-\rho_{Gn}}$. Unexpected innovations of the asset ESG score could covary with innovations in the aggregate greenness. This reflects the notion that if green firms are incentivized to become even greener, both the aggregate market and a collection of green firms would display an improved ESG profile. Conforming to intuition, estimation shows that the correlation between the market and stock level ESG is, on average, positive for green firms, negative for brown, and zero for green neutral firms. We then define the dividend growth process as

$$\Delta d_{n,t+1} = \mu_{dn} + \rho_{dn,x} x_t + \sigma_{dn,c}\varepsilon_{c,t+1} + \sigma_{dn,dM}\varepsilon_{dM,t+1} + \sigma_{dn}\varepsilon_{dn,t+1}, \tag{22}$$

where $\varepsilon_{dn,t+1}$ is IID normal and uncorrelated with all other innovations. While the idiosyncratic component of dividend growth is uncorrelated across assets, the unexpected dividend growth is allowed to covary with consumption shocks, $\varepsilon_{c,t+1}$, and market dividend shocks, $\varepsilon_{dM,t+1}$.

The following proposition characterizes the price-to-dividend ratio as well as the realized and expected return on an arbitrary risky asset.

Proposition 5. *The equilibrium price-to-dividend ratio and the dynamics of asset return* *are formulated as*

$$
\begin{aligned}
pd_{n,t} &= A_{n,0} + A_{n,G} G_{W,t} + A_{n,\delta}\delta_t + A_{n,x} x_t + A_{n,Gn} G_{n,t}, \qquad (23)\\
r_{n,t+1} &= r_{n,0} - m_G G_{W,t} + (\kappa_{n,\delta} - m_\delta)\,\delta_t - m_x x_t + \kappa_{n,Gn} G_{n,t} \\
&\quad + \sigma_{dn,c}\varepsilon_{c,t+1} + \kappa_{rn,pd} A_{n,G}\sigma_G \varepsilon_{G,t+1} \\
&\quad + \kappa_{rn,pd} A_{n,\delta}\sigma_\delta \varepsilon_{\delta,t+1} + \kappa_{rn,pd} A_{n,x}\sigma_x \varepsilon_{x,t+1} \\
&\quad + \kappa_{rn,pd} A_{n,Gn}\sigma_{Gn}\varepsilon_{Gn,t+1} + \sigma_{dn,dM}\varepsilon_{dM,t+1} + \sigma_{dn}\varepsilon_{dn,t+1}, \qquad (24)
\end{aligned}
$$

where $A_{n,G} = \frac{m_G}{1-\kappa_{rn,pd}\rho_G}$, $A_{n,\delta} = \frac{m_\delta - \kappa_{n,\delta}}{1-\kappa_{rn,pd}\rho_\delta}$, $A_{n,x} = \frac{m_x + \rho_{dn,x}}{1-\kappa_{rn,pd}\rho_x}$, $A_{n,Gn} = \frac{-\kappa_{n,Gn}}{1-\kappa_{rn,pd}\rho_{Gn}}$, $\kappa_{n,\delta} = -\frac{\bar{G}_n}{1-\bar{\delta}\bar{G}_n}$, *and* $\kappa_{n,Gn} = -\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_n}$. *The parameters* $A_{n,0}$ *and* $r_{n,0}$ *are described in Online Appendix E. The expected excess return is given by*

$$
\begin{aligned}
&\mathrm{E}_t\left[\hat{r}_{n,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[\hat{r}_{n,t+1}\right] = \\
&\quad \underbrace{\sigma_{dn,c}}_{\mathrm{Cov}_t[r_{n,t+1},\varepsilon_{c,t+1}]}\lambda_c + \underbrace{\kappa_{rn,pd}\left(A_{n,G}\sigma_G + A_{n,Gn}\sigma_{Gn,G}\right)}_{\mathrm{Cov}_t\left[r_{n,t+1},\varepsilon_{G,t+1}\right]}\lambda_G \\
&\quad + \underbrace{\kappa_{rn,pd}A_{n,\delta}\sigma_\delta}_{\mathrm{Cov}_t\left[r_{n,t+1},\varepsilon_{\delta,t+1}\right]}\lambda_\delta + \underbrace{\kappa_{rn,pd}A_{n,x}\sigma_x}_{\mathrm{Cov}_t[r_{n,t+1},\varepsilon_{x,t+1}]}\lambda_x \\
&\quad + \underbrace{\log\left(1-\bar{\delta}\bar{G}_n\right) - \frac{\bar{G}_n\left(\delta_t - \bar{\delta}\right)}{1-\bar{\delta}\bar{G}_n} - \frac{\bar{\delta}\left(G_{n,t} - \bar{G}_n\right)}{1-\bar{\delta}\bar{G}_n}}_{-y_{n,t}}, \qquad (25)
\end{aligned}
$$

where $\hat{r}_{n,t+1} = r_{n,t+1} - r_{f,t+1}$ *is the asset excess return and* $y_{n,t}$ *represents the convenience yield.*

As we have already described, in detail, the valuation ratio and the return dynamics for the market and wealth portfolios, we focus on incremental insights emerging from the cross section. Notice that the dependence of return on the market-level ESG profile, $G_{W,t}$, evolves from the risk-free rate and is identical for green and brown assets. Moreover, while the market ESG does not appear in asset excess return, the loadings on the market ESG plays an important role in explaining the cross section.

In particular, $A_{n,\delta}$ is positive for green assets, while it turns negative for assets with a sufficiently negative average ESG score.[15] Thus, the price of a green asset rises in the

[15]The condition for $A_{n,\delta}$ to be positive is $\bar{G}_n > -\frac{m_\delta}{1-m_\delta\bar{\delta}}$. The threshold is nonzero because ESG demand also drives the risk-free rate, which discounts future cashflows, and this contribution has the same sign for green and brown assets. Based on the model estimates in Section 4, we empirically verify

presence of a positive preference shock $\varepsilon_{\delta,t+1}$, driving a positive unexpected return. This is because the asset delivers higher nonpecuniary benefits in the presence of a positive preference shock. In the same vein, the price of a brown asset drops as its negative externalities are perceived stronger. Hence, preference shocks could render the green minus brown realized return spread positive, lending support to the findings in Pástor et al. (2021b), who highlight the positive association between shifts in ESG tastes over generations and unexpected returns on green assets. In addition, $G_{n,t}$, the firm ESG profile, affects stock valuation and realized return. As $A_{n,Gn} > 0$, a positive innovation $\varepsilon_{Gn,t+1}$ to the firm's ESG score entails a contemporaneous positive unexpected return.

The process $G_{n,t}$ is also the determinant of the convenience yield effect. Through the negative coefficient $\kappa_{n,Gn}$, the effect suggests a negative expected return contribution for green stocks and positive for brown. As noted earlier, the presence of convenience yield echoes the single-period setup, where investors are willing to compromise on the expected return due to holding green assets.

Analyzing the expected excess return in (25), the covariance between realized returns and ESG demand shocks is positive for green assets and negative otherwise. The expected return on green assets consists of four positive risk premia contributions, while the convenience yield effect is negative. Brown assets are characterized by positive risk premia for consumption, long-run risk, but a negative risk premium for ESG demand shocks, as well as a risk premium for aggregate ESG supply shocks that can be either positive or negative. Convenience yield is associated with a positive expected return contribution.

Thus, the expected return on a portfolio that takes long positions in green and sells short brown assets is characterized by a negative contribution due to convenience yield, and a positive risk premium due to exposures to ESG demand shocks. Our setup does not impose particular restrictions on the loadings on the other risk sources: (i) consumption $\varepsilon_{c,t+1}$, (ii) aggregate ESG supply $\varepsilon_{G,t+1}$, and (iii) long-run growth $\varepsilon_{x,t+1}$. The empirical evidence, that we discuss in detail below, shows that brown stocks have larger exposures to consumption and long-run shocks, while green stocks tend to have a slightly larger exposure to aggregate ESG supply shocks.

To summarize, the risk premium channel counters the convenience yield effect and suggests that green stocks are perceived riskier and are thus associated with higher risk premium. Thus, we reinforce the intuition from analyzing the market portfolio, that cross-sectional asset pricing implications in a dynamic setup are markedly different from

that the threshold value, $-\frac{m_\delta}{1-m_\delta \delta}$, is near zero (ESG-neutral asset). The reason is that the effect on the discount rate is negligible relative to the effect on the valuation of nonpecuniary benefits.

those in the static setup. Moreover, the important observation of Pástor et al. (2021b) that green assets have realized higher average returns that would not be expected can readily be rationalized in a dynamic setup. First, in a dynamic setup, ESG demand (δ_t) and supply ($G_{W,t}$) fluctuate and the ESG-expected return relation can go either way. Furthermore, preference shocks are associated with green assets realizing positive returns that get even higher with increasing volatility of the shocks.

Having derived the asset pricing equilibrium, we are ready to estimate the model based on data on consumption growth, as well as observations on the market portfolio and individual stock returns, dividend-to-price ratios, and ESG scores.

3 Data

We collect ESG scores from three data vendors, namely MSCI KLD (available from 1991 to 2015), MSCI IVA (available from 2007 to 2019), and Refinitiv Asset4 (available from 2002 to 2019). The choice of using scores from different providers allows to obtain a longer sample and increase firm coverage.

MSCI KLD provides periodic assessments at the firm level of a number of potential strengths and concerns related to sustainability. There are a maximum of 42 potential strengths and 29 potential concerns. The assessments are organized within categories that can be associated with the environmental, social, and governance pillars.[16] We build separate firm-level raw scores for the three pillars as the difference between (i) the total number of strengths identified, normalized by the total number of potential strengths assessed for that firm on a given date, and (ii) the total number of concerns identified, normalized by the total number of potential concerns.[17] On each observation date and per each rated firm, we calculate percentile ranks separately for environmental, social, and governance pillars and normalize the ranks between -0.5 and 0.5. We then average the three normalized ranks to obtain a total score. The total score is normalized to the range of -0.5 and 0.5. MSCI IVA provides uniformly distributed scores on a scale from 0 to 10 for the three pillars and a total ESG score. The scores do not represent an absolute

[16]The provider identifies two separate categories of assessments concerning the environment and the corporate governance. We consider the other categories of assessments, i.e., *community*, *diversity*, *employee relations*, *human rights*, and *product*, to jointly contribute to the social pillar.

[17]When the number of assessed strengths (concerns) is zero, we consider a zero contribution from strengths (concerns) to calculate the firm-specific raw score. This is relevant only for the assessment of the governance pillar between 2010 and 2015, when for some firms MSCI KLD assesses governance exclusively in terms of concerns.

assessment of a firm's ESG profile, but rather provide an assessment relative to all other firms that receive a rating on a given date. For each observation, we calculate percentile ranks of the scores and normalize the score to the range -0.5 and 0.5 for both the pillar and total scores. We follow a similar procedure for the Refinitiv Asset4 dataset, which provides pillar and total scores that are uniformly distributed on a scale from 1 to 100. We calculate a monthly firm-level consensus score by averaging among the scores available on that date.

We consider common stocks (share codes 10 and 11) traded on the NYSE/AMEX/Nasdaq exchanges, while narrowing down the focus to firms for which an ESG rating is available. Monthly total returns and market capitalization are obtained from the Center for Research in Security Prices (CRSP). We further exclude stocks that belong to the bottom percentile of market capitalization. The risk-free return is the monthly return of the 1-month Treasury Bill.[18]

We form three monthly-rebalanced portfolios through sorting on ESG scores. The brown, neutral, and green portfolios consist of stocks with consensus ESG score below the 30-th, between the 30-th and the 70-th, and above the 70-th percentile, respectively. The market ESG score is obtained by value-weighting the corresponding stock-level quantities.

To calculate the aggregate nominal consumption, we follow the existing literature (e.g., Constantinides and Ghosh, 2011, David and Veronesi, 2013, Schorfheide et al., 2018) and consider the monthly time series of personal consumption expenditures for nondurable goods and services, provided by the Bureau of Economic Analysis and available on the Federal Reserve Economic Data website (series `PCEND` and `PCES`). We then use the Personal Consumption Expenditures Price Index (series `PCEPI`) and the U.S. population (series `POPTHM`) to calculate real per capita consumption.

4 Estimation

4.1 Estimation technique

To keep the focus on incremental implications of ESG, we adopt from Bansal and Yaron (2004) several standard parameter values. In particular, the subjective discount rate is $\beta = 0.998$, the intertemporal elasticity of substitution is $\psi = 1.5$, and the parameters describing the long-run risk dynamics are $\rho_x = 0.979$ and $\sigma_x = 0.00034$. As for ESG

[18] We thank Kenneth French for making the risk-free returns available via his website: `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

demand δ_t, it is sensible to assume that the effect of shocks to ESG demand is highly persistent. However, for the solution of the model, it is useful to assume that there exists an average value $\bar{\delta}$ around which the model can be log-linearized. For this reason, we adopt the technique in Ireland (2015) and restrict the dynamics of δ_t to be near unit-root, by setting $\rho_\delta = 0.9999$.

The remaining parameter space, denoted by Θ, is composed of economy-wide parameters, Θ_E, market parameters, Θ_M, and individual asset parameters, Θ_{br}, Θ_{neu}, and Θ_{gr}, for the brown, neutral, and green portfolios, respectively. More specifically, the economy-wide parameters, denoted by $\Theta_E = \{\gamma, \mu_c, \sigma_c, x_0, \bar{G}_W, \rho_G, \sigma_G, \delta_0, \bar{\delta}, \sigma_\delta\}$, include preference parameters, short- and long-run consumption growth, aggregate greenness, and ESG demand. Market parameters, $\Theta_M = \{\mu_{dM}, \rho_{dM,x}, \sigma_{dM,c}, \sigma_{dM}\}$, underlie the dynamics of the market dividend growth in (17). Asset-specific parameters are denoted by $\Theta_j = \{\mu_{dj}, \rho_{dj,x}, \sigma_{dj,c}, \sigma_{dj,dM}, \sigma_{dj}, \mu_{Gj}, \rho_{Gj}, \sigma_{Gj,G}, \sigma_{Gj}\}$, where $j = \{br, neu, gr\}$, and they underlie the dynamics of the dividend growth and the asset greenness.

The model can be represented through a linear state space obtained by stacking the dynamics of consumption growth, aggregate ESG supply, ESG demand, long-run risk, portfolio ESG scores, the market excess return, as well as excess returns of the brown, green neutral, and green portfolios. The joint dynamics is described through the vector autoregression

$$\boldsymbol{X}_{t+1} = \boldsymbol{A}_{\boldsymbol{X}} + \boldsymbol{B}_{\boldsymbol{X}} \boldsymbol{X}_t + \boldsymbol{\sigma}_{\boldsymbol{X}} \boldsymbol{\varepsilon}_{t+1}, \tag{26}$$

where

$$\boldsymbol{X}_t = \begin{bmatrix} \Delta c_t & G_{W,t} & \delta_t & x_t & G_{br,t} & G_{neu,t} & G_{gr,t} & \hat{r}_{M,t} & \hat{r}_{br,t} & \hat{r}_{neu,t} & \hat{r}_{gr,t} \end{bmatrix}', \tag{27a}$$

$$\boldsymbol{A}_{\boldsymbol{X}} = \begin{bmatrix} \mu_c & \mu_G & \mu_\delta & 0 & \mu_{Gbr} & \mu_{Gneu} & \mu_{Ggr} & \hat{r}_{M,0} & \hat{r}_{br,0} & \hat{r}_{neu,0} & \hat{r}_{gr,0} \end{bmatrix}', \tag{27b}$$

$$\boldsymbol{\varepsilon}_t = \begin{bmatrix} \varepsilon_{c,t} & \varepsilon_{G,t} & \varepsilon_{\delta,t} & \varepsilon_{x,t} & \varepsilon_{Gbr,t} & \varepsilon_{Gneu,t} & \varepsilon_{Ggr,t} & \varepsilon_{dM,t} & \varepsilon_{dbr,t} & \varepsilon_{dneu,t} & \varepsilon_{dbr,t} \end{bmatrix}', \tag{27c}$$

and the matrices $\boldsymbol{B}_{\boldsymbol{X}}$ and $\boldsymbol{\sigma}_{\boldsymbol{X}}$ are described in Online Appendix F. The vector $\boldsymbol{X}_t$ includes variables that are unobserved, namely, the ESG demand and the long-run consumption growth. Hence, we resort to the Kalman filter to estimate the system.

The transition equation in the Kalman filter is given by (26). The observable variables are the real monthly consumption growth, the ESG score of the market and its excess return, as well as the ESG scores of the portfolios and their monthly returns. We stack these variables into the vector $\boldsymbol{Y}_t$:

$$\boldsymbol{Y}_t = \begin{bmatrix}\Delta c_t & G_{W,t} & G_{br,t} & G_{neu,t} & G_{gr,t} & \hat{r}_{M,t} & \hat{r}_{br,t} & \hat{r}_{neu,t} & \hat{r}_{gr,t}\end{bmatrix}'. \tag{28}$$

The observation equation of the Kalman filter is then given by

$$\boldsymbol{Y}_t = \boldsymbol{H}\boldsymbol{X}_t. \tag{29}$$

Further details on the state-space representation, including the components of the $\boldsymbol{H}$ matrix and the Kalman filter implementation, are provided in Online Appendix F.

In order to (i) address the finite-sample properties of the Kalman filter based maximum likelihood estimates and (ii) impose constraints on the parameter space (e.g., δ_t is constrained to be nonnegative and further $\bar{\delta}$ is set to be equal to the sample mean of δ_t), which could render asymptotic inferences unreliable, we implement the methodology proposed by Efron and Tibshirani (1994, Ch. 6) and employed by Ireland (2015). In particular, we simulate 1000 joint trajectories of the variables in the model using the point estimates of the parameters. For each trajectory, we re-estimate the model based on the simulated data and, finally, we evaluate the standard errors per parameter as the standard deviations across all estimated values.

4.2 Estimating the model

We first present the time series dynamics of aggregate ESG demand and supply (both in Figure 2). Estimated through Kalman Filter, the ESG demand, δ_t, reveals similar patterns to those characterizing the press attention to sustainable investing (Figure 1b). In particular, δ_t increases towards the beginning of the 90s, then shows a diminishing trend until 2010s, and finally advances, quite sharply, between 2015 and 2020.[19] When the sample ends, δ_t reaches a value equal to 0.1%, about three times larger than the sample mean $\bar{\delta} = 0.035\%$. Thus, for an asset with a given ESG score, the ESG based convenience yield at the end of the sample is also three times larger than its average value.

The aggregate ESG supply, $G_{W,t}$, represents the value-weighted market ESG score. The ESG supply is highly persistent. Unsurprisingly, ESG supply is less volatile than ESG demand, as demand is driven by market sentiment, which could be subject to sudden

[19] We verify that the innovations of δ_t, i.e. $\varepsilon_{\delta,t}$ in equation (10), are not serially correlated, as all autocorrelation coefficients are insignificant.

shifts. In addition, reallocation of capital across financial assets can be done quickly and with low costs. Supply, on the other hand, is less flexible as it represents the firm's core business environment, which is quite persistent, while the costs of adjustment can be prohibitively high. As the ESG scores range between -0.5 and 0.5, the market portfolio only modestly departs from green neutrality, on average. However, the market becomes substantially more green towards the second half of the sample.[20] Profit maximizing corporations respond to ESG trends for various reasons including tax benefits, innovation stimulus, access to loans and grants, reduced cost of capital, and the attempt to cater to customers who prefer corporations with a strong enough environmental reputation. Barnett et al. (2020) also show that the increasing social cost of carbon emissions leads firms to reduce their environmental impact in equilibrium.

The estimates for the model parameters are reported in Table 1. The estimated risk aversion, γ, is 12.11, insignificantly different from 10, the value considered by Bansal and Yaron (2004). The risk aversion estimate is also within the confidence intervals of previous estimates (e.g., Constantinides and Ghosh, 2011; Schorfheide et al., 2018). The persistence of aggregate grenness, described in (9), is quite strong with ρ_G above 0.99. The volatility σ_G is 0.01 and $\bar{G}_W$, the sample mean, is 0.04. The mean and the volatility of the ESG demand are $\bar{\delta} = 0.00035$ and $\sigma_\delta = 0.00005$.

The values of $\bar{G}_W$ and $\bar{\delta}$ can be used to evaluate the average ESG benefits formulated in (2). In particular, notice that the long-run ratio between investable wealth and monthly consumption is about 480.[21] When the state variables are equal to their sample means, the consumption bundle A_t equals the current value of consumption C_t times the expression $1 + \bar{\delta}\bar{G}_W e^{\overline{pc}}$ $(= 1.0074)$. Thus, the ESG nonmonetary benefits amount to 0.74% of the physical consumption, which is significant at conventional levels. While this value reflects the entire sample period, ESG benefits rise to 5.00% of physical consumption in 2020 due to advancing levels of ESG demand. Moreover, considering a growth in ESG demand

[20] Our assessment of the market greenness could be conservative. This is because, consistent with ESG raters who provide only a relative assessment of the sustainability profile of any rated firm, the median ESG score across stocks is zeroed out at any time period in the sample. Thus, possible shifts towards sustainability that could characterize the median corporation in the sample do not come to play. The market ESG score still fluctuates, with a substantial upward trend between 2003 and 2013, because bigger market cap firms tend to be more sustainable than their smaller cap counterparts

[21] The mean logarithmic ratio (investable wealth divided by the monthly consumption, $\overline{pc}$) is obtained, based on equation (12), as $\overline{pc} = A_{pc,0} + A_{pc,G}\bar{G}_W + A_{pc,\delta}\bar{\delta}$. The quantities $A_{pc,0}$, $A_{pc,G}$, and $A_{pc,\delta}$ (presented in Online Appendix B) can be evaluated using the estimated model parameters in Table 1, as well as further substituting $\bar{G}_W = 0.04452$ and $\bar{\delta} = 0.00035$. It thus follows that $\overline{pc} = 6.17$, corresponding to a linear ratio $e^{\overline{pc}}$ of about 480. This monthly figure is translated into a ratio of 40 when consumption is measured annually.

equal to the realized value over the 2010 through 2020 period (and setting the market ESG score to be equal to the end of sample value), the expected ESG benefits would rise to 8.62% in 2030.[22]

Moving to the market prices of risk, as derived in Proposition 2, the risk premia λ_c, λ_G, λ_δ, and λ_x depend on the economy-wide parameters. Consistent with the inferences made in the theory section, we confirm that the prices of risk are all positive and statistically significant.

Consistent with the literature on long-run risk, the expected market dividend growth in (17) is leveraged relative to the expected consumption growth through the coefficient $\rho_{dM,x}$, which is equal to 3.36 and highly significant. This implies that the market portfolio is strongly exposed to the long-run risk variable. Focusing on the dividend process of ESG-sorted portfolios, described in equation (22), the expected dividend growth of the brown portfolio loads more heavily on expected consumption growth ($\rho_{dbr,x} = 3.58$) than the green portfolio ($\rho_{dgr,x} = 3.34$), with a p-value for the difference lower than 0.05.

Thus, brown stocks are more exposed to long-run cashflow risk. Likewise, brown stocks also display a higher exposure to short-run consumption risk, as the difference between $\sigma_{dbr,c} = 0.00383$ and $\sigma_{dgr,c} = 0.00123$ is significant. These findings are in line with the existing literature, documenting that a better ESG profile reduces the risk exposure (e.g., Hoepner et al., 2019; Ilhan et al., 2021). Intuitively, a higher standard of corporate ESG practice helps mitigate the legal, regulatory, and operational risks. In addition, Albuquerque et al. (2019) show that firms with a high corporate social responsibility score display higher profit margins, less cyclical profits, and lower systematic risk.

The portfolio ESG scores are highly persistent, as the parameter ρ_{Gn} in equation (21) is 0.97 or above for all three portfolios. Thus, a shock to the ESG score of an asset is expected to have long-lasting price effects, as we further analyze in Section 5. The correlation between innovations in the ESG scores of the green portfolio and the market, measured through $\sigma_{Ggr,G}$, is positive and significant, while a negative and significant (although weaker) correlation applies to the brown portfolio. The overall evidence suggests that greener assets have sustainability profiles that are more positively correlated with the aggregate ESG supply shocks.

[22]Based on equation (2), the ratio between ESG benefits and physical consumption is $\delta_t G_{W,t} e^{pc_t}$, where $pc_t = \log \frac{W_t - C_t}{C_t}$ is given in equation (12). The ratio is evaluated using the filtered time series of the state variables. The 2020 figure is obtained as the average value over the most recent 12 monthly observations. The 2030 projection is based on a linear growth of the ESG demand δ_t between 2020 and 2030 at the same rate as that realized between the 2010 and the 2020. The other state variables, namely $G_{W,t}$ and x_t, are set at the 2020 monthly average values.

We turn to analyze asset pricing moments, reported in annualized terms in Table 2. The average annual excess return of the market portfolio is 8.05% over the entire sample period (12.27% between 2018 and 2020), while both the green and brown portfolios have average excess returns of 8.26% over the sample (14.21% for the green and 7.01% for the brown portfolios between 2018 and 2020). Below, we describe the conflicting forces underlying the prices of green and brown assets and we analyze the gap between expected and realized values. Special attention is paid to the most recent years, when greener assets have realized materially higher returns.

Starting with the equity premium, it is characterized by multiple determinants, while the model provides a clear decomposition mechanism through equation (20). The market premium can mostly be attributed to long-run risk shocks (8.38%) and short-run consumption shocks (0.19%). The ESG based components of the market premium, namely the ESG supply risk premium, the ESG demand risk premium, and the average convenience yield, are all near zero, as the market portfolio only modestly departs from green neutrality, on average.

In the recent sample period, the convenience yield contribution becomes more meaningful, as the ESG demand and supply considerably exceed their sample averages. The convenience yield premium, based on the most recent 36 monthly observations (between 2018 and 2020), is significant at -6 basis points, while the average value in 2020 is -12 basis points. Moreover, considering a linear growth of ESG demand at the same rate as that realized between 2010 and 2020, while the market ESG score is kept unchanged, the expected convenience yield premium will be -21 basis points in 2030.

Increasing ESG demand in recent years is associated with significant implications for the realized market return. In particular, we report the sum of the expected excess market return and the average unexpected market return due to ESG demand shocks, i.e., the annualized average of the expression $\kappa_{rM,pd} A_{M,\delta} \sigma_\delta \varepsilon_{\delta,t+1}$ in equation (19). As the market is near ESG neutral over the entire sample, the component of realized market returns induced by unexpected shocks to ESG demand amounts to only a few basis points per annum. However, considering the most recent 36 monthly observations between 2018 and 2020, the average expected excess market return is 7.43%, while further accounting for unexpected shocks to ESG demand, the excess market return becomes 8.08%. Thus, the unexpected increase in ESG demand, over recent years, has an incremental effect of more than 50 basis points on the market return. Evidence is consistent with Pástor et al. (2021b), who highlight that ESG demand shocks have a substantial effect on contemporaneously realized asset returns.

We move on to analyze the decomposition of ESG portfolio expected excess returns, as described in equation (25). Relative to the green portfolio, the brown has a higher risk premium associated with short-run consumption shocks (0.51% vs 0.17%) and long-run shocks (9.11% vs 8.33%). Per equation (25), the ESG supply risk premium depends on the positive term $\kappa_{rn,pd} A_{n,G} \sigma_G$ and the term $\kappa_{rn,pd} A_{n,Gn} \sigma_{Gn,G}$, which is positive for the green and negative for the brown portfolios. While the first term dominates, the overall contribution of ESG supply risk premium to the expected return is negligible for all portfolios, as the volatility of aggregate and individual asset ESG scores is modest. In contrast, consistent with a positive coefficient $A_{n,\delta}$ for green stocks in (25), the ESG demand risk premium is positive and economically significant for the green portfolio (0.16% per annum), while it is -0.16% for the brown (negative $A_{n,\delta}$ coefficient). This implies a positive and statistically significant 32-basis point ESG demand risk premium for the green-minus-brown portfolio.

The ESG demand risk premium offsets the convenience-yield based effect, which amounts to 28-basis point negative contribution to the green-minus-brown expected return spread. While the ESG demand premium marginally dominates, the convenience yield effect shows substantial fluctuations over the sample period. In the presence of its current trend, the convenience yield based premium is likely to dominate. For instance, in 2020, the convenience yield premium of the green-minus-brown portfolio amounts to -60 basis points per annum, more than twice the average value over the sample. Considering a similar growth in ESG demand as that realized between 2010 and 2020, the convenience yield gap would be around one percent per annum in 2030.

Overall, the model-implied average expected excess return of the green portfolio is 7.38%, while it is higher at 8.29% for the brown portfolio. The green-minus-brown portfolio has a negative and statistically significant average expected return of -0.91% per annum. However, throughout the sample, the unexpected shocks to ESG demand induce a positive unexpected return that adds to the conditional expected return, as reflected by the term $\kappa_{rn,pd} A_{n,\delta} \sigma_\delta \varepsilon_{\delta,t+1}$ in equation (24). Considering the combined effect of the conditional expected return and the unexpected return due to demand shocks, the green-minus-brown portfolio average return is minor at -0.05% and insignificant, consistent with the negligible spread observed in the data. In the short run, however, the effect of shifts in tastes for ESG investing can play a meaningful role on the realized return of the green-minus-brown portfolio. For instance, between 2018 and 2020, the average conditional expected return of the green-minus-brown portfolio is negative at -0.97% per annum, while the model-implied annual return accounting also for ESG demand shocks is 8.39%, close to the realized value of 7.19%.

Our model highlights the notion that the conditional expected and the realized return of the green-minus-brown portfolio depends on several forces. The model also provides a structural description of the relation between unexpected shifts in tastes for ESG and realized returns. The impact of unanticipated ESG demand shocks on realized returns can be very sizable, calling for caution when inferring future returns of ESG investments based on past realized returns. In the first, the presence of ESG shocks represent a relatively small part of the sample. If anything, looking forward, the expected return on green stocks should diminish with the growing convenience yield. Hence, when positive preference shocks attenuate in the future, green assets are expected to deliver lower performance.

We conclude that the presence of time-varying convenience yield and the offsetting forces on expected and realized returns could explain the mixed evidence in the literature on return predictability of ESG ratings (e.g., Gompers et al., 2003; Hong and Kacperczyk, 2009; Edmans, 2011; Pedersen et al., 2021; Bolton and Kacperczyk, 2021).

4.3 Time-series implications

We analyze the time series dynamics of the state variables and their implications for expected and realized returns. The top-left panel in Figure 3 shows the expected consumption growth, represented by the conditionally deterministic component $\mu_c + x_t$ in (7), where x_t is obtained through the Kalman filter. The expected consumption growth is highly persistent and fluctuates between 0% and 5% per annum for most of the sample, consistent with the existing literature (e.g., Schorfheide et al., 2018). Two exceptions are the 2002 stock market crash and the 2008 financial crisis, when the expected consumption growth turns negative.

The ESG scores of the three portfolios, shown in the top-right panel, are highly persistent and they do not show clear patterns over time. In our model, the expected excess return of an ESG-neutral asset is time invariant, as the convenience yield component in (25) is constant. Indeed, in the second graph on the left of Figure 3, the green neutral portfolio displays expected excess return that is nearly constant.[23] The market premium slightly falls during the end of the sample. The reason is that the market becomes greener and therefore its convenience yield increases, especially when ESG demand grows consid-

[23]Due to value weighting and changing composition in stocks belonging to the green neutral portfolio, the expected return shows some, albeit minor, fluctuations throughout the sample period.

erably higher in the most recent years, as shown in Figure 2. As the market convenience yield rises (second graph from top on the right), the equity premium diminishes.

The green and brown portfolios show opposing patterns of expected returns due to convenience yield, which is positive for green stocks and negative otherwise. Interestingly, the green portfolio has a similar or slightly higher expected return than the green neutral portfolio for most of the sample. Despite the negative convenience yield effect on expected return, green assets deliver a positive risk premium for the exposure to ESG demand shocks (16 basis points in Table 2). For green assets, the total ESG premium, i.e., the net effect of convenience yield, ESG demand, and ESG supply risk premia, often amounts to higher returns relative to green neutrality, as displayed in the bottom-left panel of Figure 3.

The bottom-right graph displays the price-to-dividend ratios of the market portfolio, as well as those of ESG-sorted portfolios, which are highly correlated and mostly driven by the expected consumption growth variable, x_t. However, focusing on the ESG portfolios, it can be noticed that the model-implied price-to-dividend ratio of the green portfolio is lower than that of the other portfolios when ESG demand is low, while it can grow higher for increasing values of δ_t. This is because as the ESG demand rises, there is a contemporaneous positive (negative) price pressure on green (brown) assets, as displayed in equation (23).

The contemporaneous price effect of ESG demand shocks has a sizable impact on realized returns, as displayed in equation (24). In particular, green assets realize returns that are positively correlated with ESG demand shocks, while brown asset returns are negatively correlated. Figure 4 highlights this effect, with the left panel showing the cumulative realized return of the green-minus-brown portfolio, the cumulative conditional expected return, i.e., the deterministic component in equation (24), as well as the cumulative conditional expected return augmented by the demand shocks based return, as expressed through $\kappa_{rn,pd} A_{n,\delta} \sigma_\delta \varepsilon_{\delta,t+1}$ in the same equation.

As the conditional expected return of the green-minus-brown portfolio is negative throughout the sample, the cumulative expected return is negative and decreasing with time. The cumulative realized return strongly departs from the expected pattern, and more so when the variable δ_t in Figure 3 displays large shocks. This is particularly evident at the end of the 90s as well as in the most recent years of the sample, when positive shocks to ESG demand generate highly positive realized returns.

As shown in the graph, the additional effect of shocks to δ_t, when combined with the conditional expected return contribution, is crucial in explaining the realized returns of the green-minus-brown portfolio. Overall, throughout the 29-year long sample, the net increase in δ_t makes the realized return of the spread portfolio to be around zero, even in the presence of a negative cumulative conditional expected return of about 28%. The right graph in Figure 4 shows the conditional expected and realized returns accumulated over the prior 12 months. While the conditional expected return is slightly negative and shows little time variation, the 12-month realized return and the expected return augmented by the effect of ESG demand shocks are rather volatile and strongly correlated. The time-series evidence reinforces the notion that ESG demand plays a crucial role in determining realized returns of assets with ESG profiles that depart from green neutrality. In the short run, the unexpected contribution to realized returns induced by innovations to δ_t can markedly dominate the expected return component.

5 Aggregate shocks to ESG demand and supply

We next examine the asset pricing implications of ESG demand and supply shocks.

The left graphs in Figure 5 focus on demand shocks. At time $t = 0$, the state variables are set equal to their sample averages. Then, a one-standard deviation positive annual shock is applied to ESG demand.[24]

It is evident from the first graph that due to persistent ESG demand ($\rho_\delta = 0.9999$), δ_t rises and remains nearly fixed at the post-shock level. The second graph shows the conditional expected excess return of the green and the brown portfolios. The brown portfolio has a higher expected return and the gap even widens with positive ESG demand shocks due to the increasing convenience yield of green stocks, per equation (25).

The third graph shows the monthly realized portfolio excess returns. During the shock, the contemporaneous positive (negative) effect of an unexpected increase of ESG demand on realized returns of the green (brown) portfolio is sizable. This contribution is assessed based on the term $\kappa_{rn,pd} A_{n,\delta} \sigma_\delta \varepsilon_{\delta,t+1}$ in equation (24), where $A_{n,\delta}$ is positive for the green portfolio and negative otherwise. Green assets display a realized monthly return that is more than 0.5% higher than brown assets. After the end of the shock ($t = 12$ months), the realized return spread of the green-minus-brown portfolio drifts back to -9 basis points, the conditional expected return.

The cumulative return of the spread portfolio (fourth graph) steeply increases during

[24] As the frequency used for the model estimation is monthly, the size of a one-standard deviation annual shock to δ_t is $\sigma_\delta\sqrt{12}$. Thus, we impose 12 positive consecutive monthly shocks of size $\frac{\sigma_\delta\sqrt{12}}{12}$ at months $t = 1, \ldots, 12$.

the shock, reaching about 6% at the end of the 1-year shock, and then, when the shock is shut down, slowly diminishes due to the convenience yield effect. The positive effect of realized returns vanishes about 72 months following the end of the shock.

The experiment highlights that while the expected green-minus-brown portfolio return is negative, unexpected positive ESG demand shocks have a substantial contemporaneous effect on realized returns. The contemporaneous effect of ESG demand shocks is also evident from the valuation ratios, per equation (23). The green portfolio price-to-dividend ratio reflects the reduced expected cost of capital, following ESG demand shocks, hence rising from 56 to 58. The brown portfolio displays a negative price effect.

The graphs on the right reflect the effects of a positive shock to aggregate ESG supply. The size of the annual unanticipated shock is +0.1, equally distributed throughout the 12 months, reflects the ESG profile improvement of one decile on a scale ranging from the most brown to the most green. The aggregate ESG supply is less persistent ($\rho_G = 0.97$) than the aggregate demand, consequently, the effect of the shock vanishes, albeit rather slowly.

To understand the effect of an ESG supply shock on expected returns (second graph from top), it is important to recall that the ESG score of the green (brown) portfolio is positively (negatively) correlated with ESG supply, as $\sigma_{Ggr,G}$ in (21) is positive and $\sigma_{Gbr,G}$ is negative. Hence, with a positive aggregate shock, the convenience yield of the green asset increases and the expected excess return diminishes per equation (25). The opposite applies to brown assets. The effect of ESG supply shocks is altogether milder relative to demand shocks.

Consistent with equation (24), a positive shock to aggregate ESG implies a positive unexpected return, per the term $\kappa_{rn,pd} A_{n,G} \sigma_G \varepsilon_{G,t+1}$ with positive $A_{n,G}$, as well as an indirect effect due to the correlation of the shock with the asset ESG score, per the term $\kappa_{rn,pd} A_{n,Gn} \sigma_{Gn,G} \varepsilon_{G,t+1}$ with positive $A_{n,Gn}$. The first contribution depends on the negative effect on the risk-free rate, as displayed in equation (15), which implies a higher valuation of future cashflows. The second component reflects the change of the asset convenience yield in the presence of brown aversion, which is positive for the green ($\sigma_{Gn,G} > 0$) and negative for the brown ($\sigma_{Gn,G} < 0$) portfolios. The net effect is positive for both portfolios, as the risk-free rate effect dominates the convenience yield effect, and it is stronger for the green portfolio. The size of the unexpected return induced by an ESG supply shock is significantly smaller than that of the ESG demand shock. Also, while the unexpected return due to the ESG supply shock is larger for the green portfolio, the realized return of the green-minus-brown portfolio is still uniformly negative, reflecting the negative expected return gap.

Finally, following ESG supply shock, the price-to-dividend ratio of both portfolios increases due to the lower discount rate, per the term $A_{n,G}G_{W,t}$ in (23). The green portfolio experiences a larger price increase than the brown portfolio, as the contemporaneous positive revision of the portfolio ESG score implies a positive price effect, per the term $A_{n,Gn}G_{n,t}$, which adds to that of a diminishing risk-free rate effect.

In the baseline analysis, we assume that the asset dividend growth in (22) is uncorrelated with innovations in both ESG demand and ESG scores. In the Online Appendix, we provide supplementary analyses relaxing that assumption. An unexpected increase in ESG demand may, for instance, reinforce demand for green products, thus boosting the profits of green firms on the account of brown firms. The graphs on the left in Figure A.1 in the Online Appendix show that, when dividend growth is positively correlated with ESG demand, the expected return of the green-minus-brown spread portfolio increases relative to the zero correlation case, while a negative correlation implies a lower expected return. This is because the positive correlation implies a return contribution that is also positively correlated with ESG demand, and thus a higher loading on the positive price of risk of ESG demand. If the green (brown) asset's dividend growth is positively (negatively) correlated with ESG demand, during a positive ESG demand shock the positive realized return gap in favor of the green asset widens, while the equilibrium expected return gap in favor of the brown asset shrinks. In this case, the positive effect on the cumulative return of the green-minus-brown portfolio is stronger and vanishes over a longer period relative to the zero correlation case.

The graphs on the right in Figure A.1 show the response to an annual shock to the ESG score of the green asset. When the correlation between the dividend growth and the ESG score is zero, the effects on returns are qualitatively similar to those described for an aggregate ESG demand shock, while the impact on both expected and realized returns is slightly weaker. This is because a positive shock to the ESG score triggers only an increase in the convenience yield, but does not imply a reducing risk-free rate.

A positive correlation between dividend growth and ESG score is plausible if an improvement of the firm's ESG profile triggers a higher demand for goods and services and thus higher cashflows. Then, the realized return corresponding to the positive ESG score shock can be significantly higher than that in the baseline case. The correlation between dividend growth and ESG score could yet be negative. For instance, this could result from increasing costs incurred for the improvement of the firm's sustainability profile. Then, the negative cashflow effect could imply a lower, even negative, realized return due to the unexpected ESG score improvement.

6 Conclusion

While asset pricing with impact investing has thus far focused on single-period setups, we study the dynamic effects through a general equilibrium model. The economic agent in the model is ESG perceptive, deriving utility from both consuming goods and services as well as holding green assets. Asset ESG scores (supply) and the demand for greenness are stochastic and persistent. In equilibrium, there are two incremental risk factors evolving from aggregate ESG preference shocks and supply shocks.

As green assets positively load on the demand and supply shocks, they command a higher risk premium compared to brown. On the other hand, green assets are also associated with time-varying positive convenience yield, a force leading to lower expected return. The expected return gap between green and brown assets is thus inconclusive. However, preference shocks could make green assets realize unexpected returns, so that the green-minus-brown realized return is positive and large.

We filter out the latent state of greenness demand and demonstrate its time variation. Notably, ESG demand displays a substantial upward trend in the most recent years. Evidence shows that ESG preference shocks represent a novel risk source characterized by positive premium, while supply shocks are second order. The estimates also show that the positive green-minus-brown cumulative realized return can last for up to six years following an annual one standard deviation preference shock. Finally, the non-monetary benefits associated with sustainable investing account for a nontrivial fraction of total consumption that is expected to grow.

We offer avenues for future research. First, while we focus on dynamic equilibrium, we leave it for future work to study the asset allocation across characteristics sorted portfolios in the presence of ESG preference shocks. Then, our model could be extended to account for heterogeneity in ESG preferences as well as heterogeneous beliefs about the firm's ESG profile. Finally, our findings suggest that the interaction of time-varying preferences for impact investing and ESG characteristics could be a useful driver of equity demand per Koijen and Yogo (2019).

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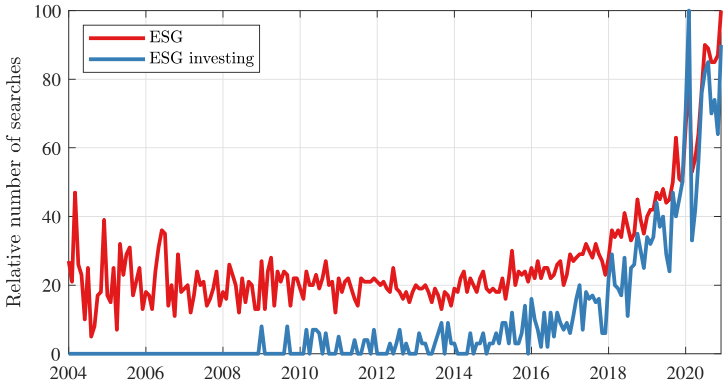
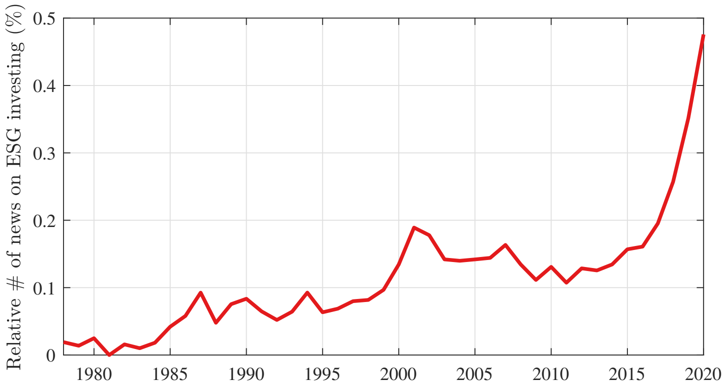
Figure 1: Trends in ESG investing

Panel (a) shows the Google Trends data from January 2004 to December 2020 for web searches in the U.S. of the terms "ESG" and "ESG investing". The popularity score is calculated monthly as the total number of searches for the specified string divided by the total number of Google searches in the selected area. The time series is then scaled between 0 and 100. Panel (b) shows the number of Factiva newspaper articles in English language on ESG/sustainable/socially responsible/ethical investing/investment, relative to the total number of news on investing/investment. Panel (c) shows the yearly money flows into U.S. sustainable funds from 2009 to 2020, as well as the fraction of assets under management of U.S. sustainable funds relative to the entire U.S. mutual fund industry (data from Morningstar).

(a) Popularity of Google web searches in U.S. for the terms "ESG" and "ESG investing"



(b) Number of articles on ESG investing relative to total number of articles on investing



(c) U.S. sustainable fund flows

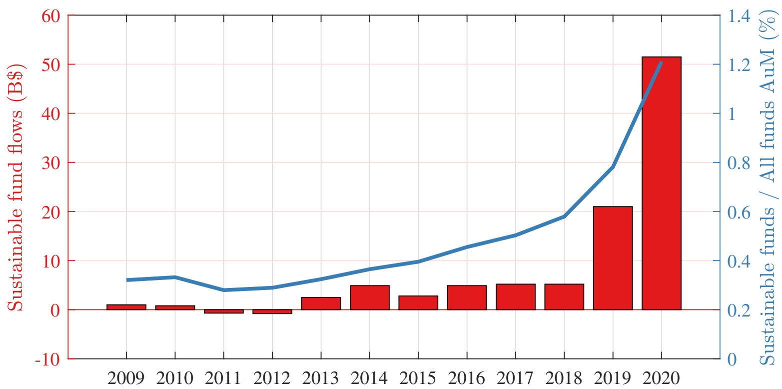


Figure 2: Time series of aggregate ESG demand and supply.

The figure shows the estimated time series of aggregate ESG demand, δ_t, and supply, $G_{W,t}$. The sample runs from 1991 to 2020.

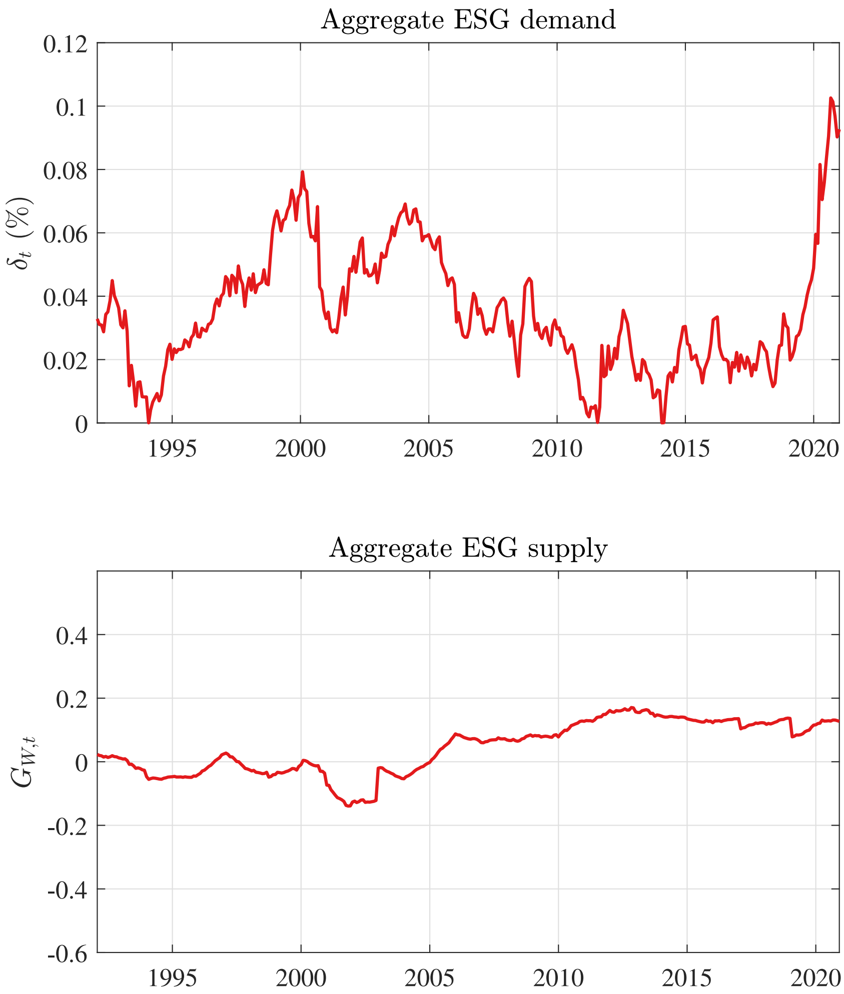


Figure 3: Time series of expected consumption growth, ESG scores, expected excess returns, and price-to-dividend ratios.

The figure shows the estimated time series of expected consumption growth, ESG scores, expected market and portfolio excess returns, convenience yields from ESG investing, total ESG premia, as well as price-to-annual dividend ratios. All quantities are annualized. The green, neutral, and brown portfolios are obtained sorting stocks by consensus ESG score. The estimation is performed by maximum likelihood, observing the time series of market and portfolio returns, consensus ESG ratings, and consumption growth, as well as average price-to-dividend ratios. The sample runs from 1991 to 2020.

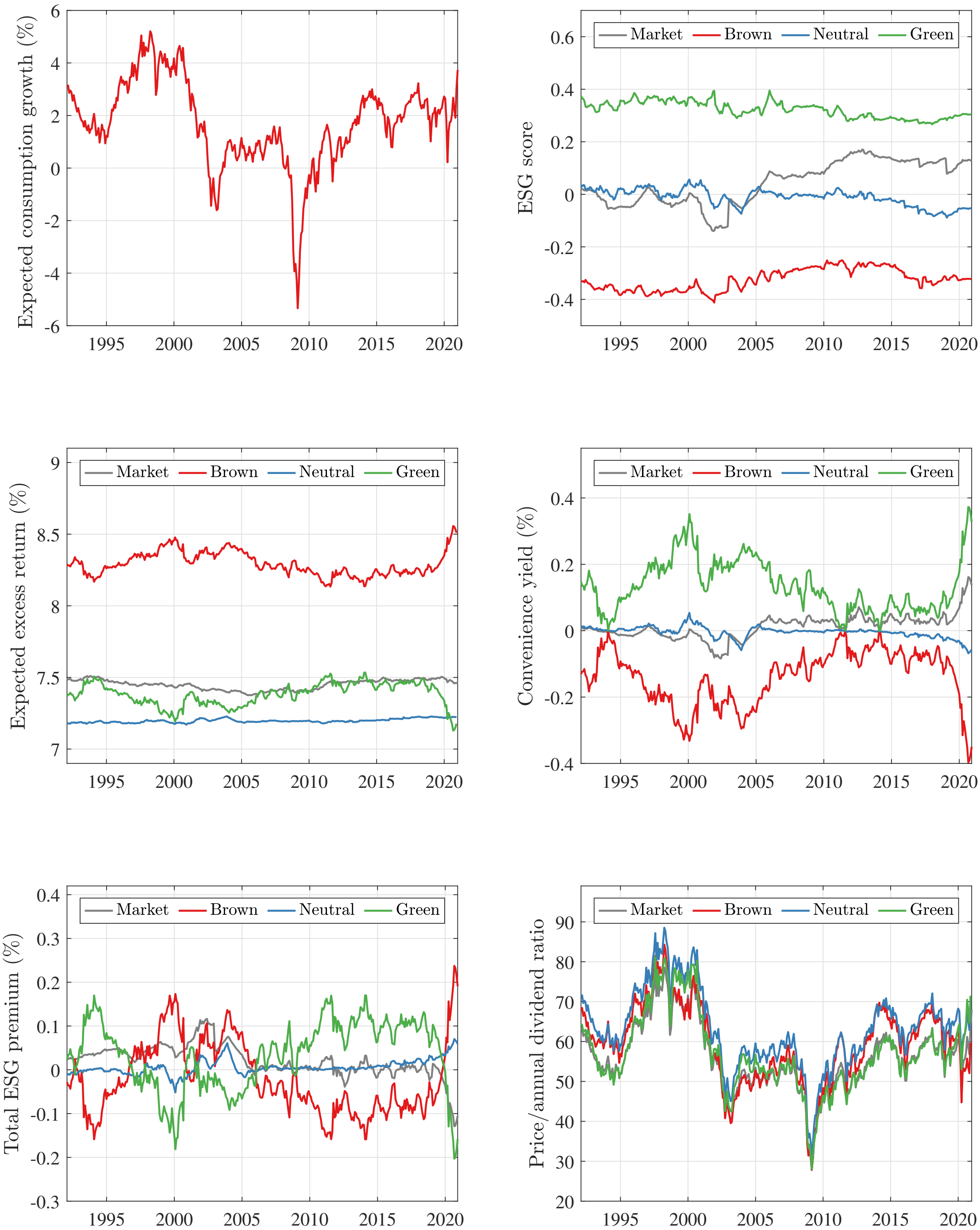
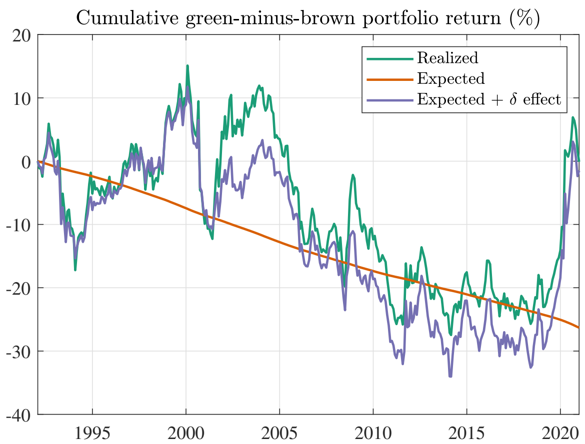


Figure 4: Returns of green-minus-brown portfolio.

The left graph shows the realized cumulative logarithmic return of the green-minus-brown portfolio, as well as the model-implied expected return and the model-implied expected return accounting for the unexpected contribution driven by the innovations in the preference state variable δ_t. The right graph shows the 12-month rolling logarithmic return of the same portfolio. The estimation is performed by maximum likelihood, observing the time series of market and portfolio returns, consensus ESG ratings, and consumption growth, as well as average price-to-dividend ratios. The sample runs from 1991 to 2020.



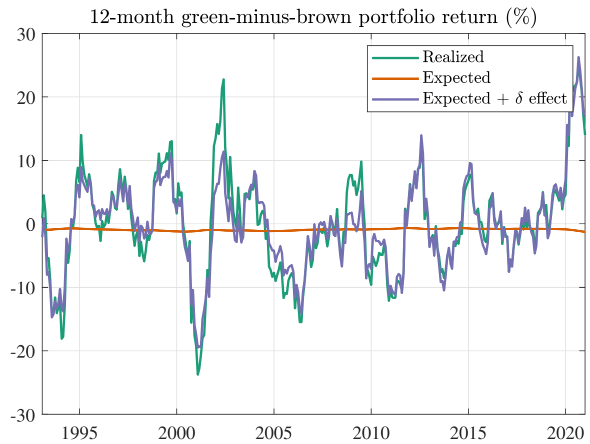


Figure 5: Response to annual one-standard deviation shock to aggregate ESG demand and supply.

The graphs on the left show responses to a one-standard deviation positive annual shock applied to δ_t. The graphs on the right show responses to a $+0.1$ annual shock to aggregate greenness. The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months.

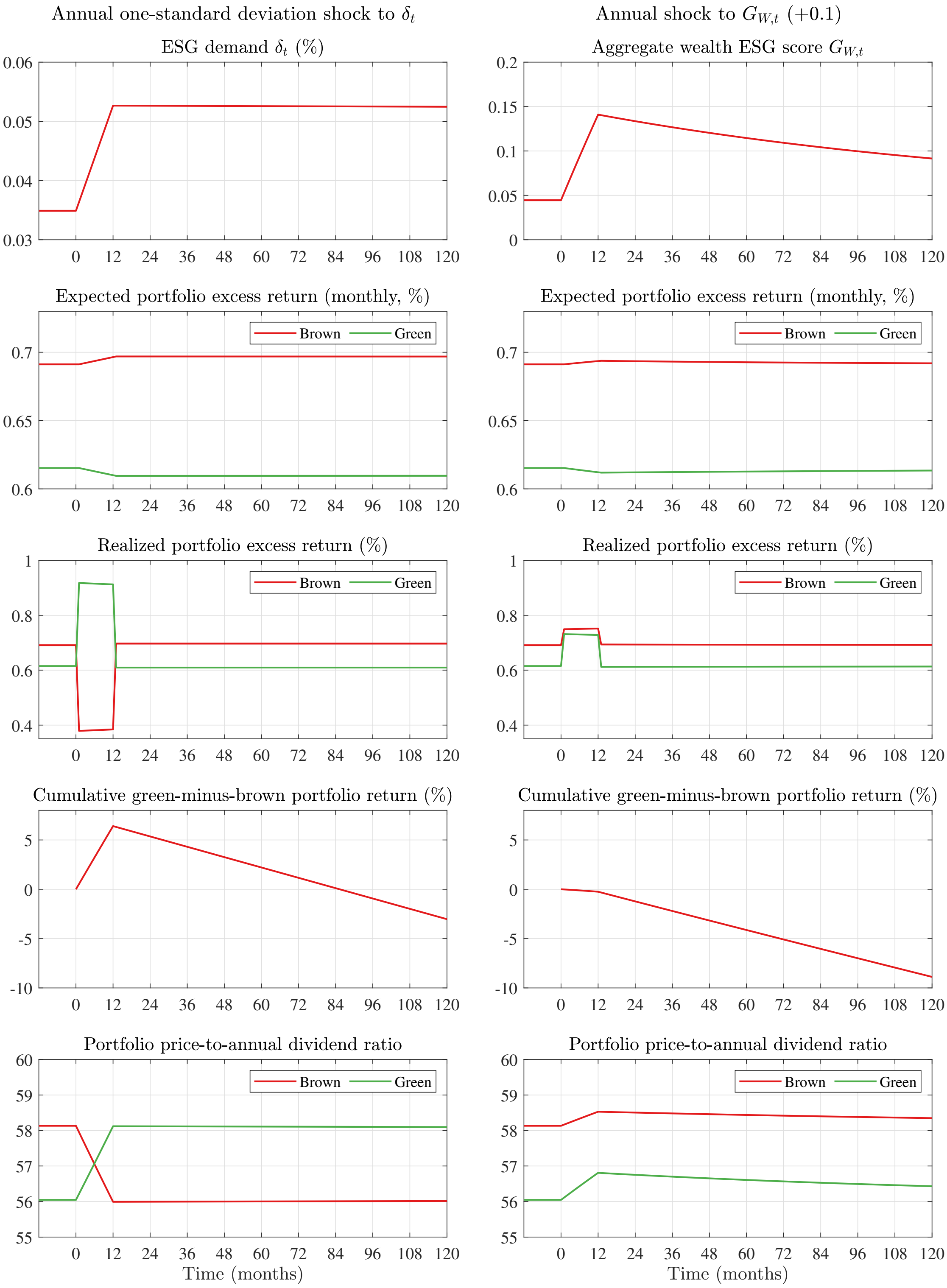


Table 1: Parameter estimates.

The table reports the estimated parameters for the baseline model specification. The subjective discount rate β is set at 0.998, the intertemporal elasticity of substitution ψ at 1.5, the long-run risk persistence ρ_x at 0.979, its volatility σ_x at 0.00034, and the persistence of ESG demand ρ_δ at 0.9999. The brown, neutral, and green portfolios are obtained by value-weighting stocks sorted by consensus ESG score. The estimation procedure is described in Section 4.1. The sample runs from 1991 to 2020.

Economy-wide parameters (Θ_E) and market prices of risk

γ	μ_c	σ_c	x_0	μ_G	ρ_G	σ_G	δ_0	$\bar{\delta}$	σ_δ
12.11057	0.00141	0.00925	0.00118	0.00030	0.99337	0.01064	0.00033	0.00035	0.00005
(1.11048)	(0.00059)	(0.00036)	(0.00210)	(0.00029)	(0.00641)	(0.00040)	(0.00015)	(0.00013)	(0.00000)
λ_c	λ_G	λ_δ	λ_x						
0.11199	0.00607	0.01267	0.17005						
(0.01099)	(0.00204)	(0.00154)	(0.01644)						

Market portfolio parameters (Θ_M)

μ_{dM}	$\rho_{dM,x}$	$\sigma_{dM,c}$	σ_{dM}
0.00637	3.35832	0.00144	0.01267
(0.00084)	(0.26521)	(0.00222)	(0.00798)

Brown portfolio parameters (Θ_{br})

μ_{dbr}	$\rho_{dbr,x}$	$\sigma_{dbr,c}$	$\sigma_{dbr,dM}$	σ_{dbr}	μ_{Gbr}	ρ_{Gbr}	σ_{GbrG}	σ_{Gbr}
0.00714	3.58388	0.00383	0.00728	0.00181	-0.00369	0.98863	-0.00840	0.01683
(0.00076)	(0.22522)	(0.00243)	(0.00821)	(0.00229)	(0.00331)	(0.01020)	(0.00099)	(0.00065)

Neutral portfolio parameters (Θ_{neu})

μ_{dneu}	$\rho_{dneu,x}$	$\sigma_{dneu,c}$	$\sigma_{dneu,dM}$	σ_{dneu}	μ_{Gneu}	ρ_{Gneu}	σ_{GneuG}	σ_{Gneu}
0.00632	3.34334	0.00061	0.02067	0.00455	-0.00033	0.96928	-0.00076	0.00730
(0.00113)	(0.37589)	(0.00241)	(0.01089)	(0.00220)	(0.00015)	(0.01414)	(0.00039)	(0.00028)

Green portfolio parameters (Θ_{gr})

μ_{dgr}	$\rho_{dgr,x}$	$\sigma_{dgr,c}$	$\sigma_{dgr,dM}$	σ_{dgr}	μ_{Ggr}	ρ_{Ggr}	σ_{GgrG}	σ_{Ggr}
0.00633	3.33846	0.00123	0.00859	0.00725	0.00183	0.99434	0.01062	0.01440
(0.00076)	(0.22033)	(0.00222)	(0.00741)	(0.00055)	(0.00231)	(0.00717)	(0.00084)	(0.00054)

Table 2: Annualized moments and excess return decomposition.

The table reports the observed and model-implied annualized moments. The brown, neutral, and green portfolios are obtained by value-weighting stocks sorted by consensus ESG score. The estimation procedure is described in Section 4.1. The sample runs from 1991 to 2020.

Portfolio	Market	Brown	Neutral	Green	Green-minus-brown
Data					
Average excess return (full sample)	8.05%	8.26%	7.94%	8.26%	-0.00%
Average excess return (2018–2020)	12.27%	7.01%	11.01%	14.21%	7.19%
Excess return volatility	14.82%	16.14%	15.98%	15.04%	8.06%
Model					
Short-run consumption risk premium	0.19%	0.51%	0.08%	0.17%	-0.35%
	(0.31%)	(0.33%)	(0.34%)	(0.31%)	(0.17%)
Long-run consumption risk premium	8.38%	9.11%	8.39%	8.33%	-0.79%
	(0.94%)	(0.82%)	(1.31%)	(0.80%)	(0.15%)
ESG supply risk premium	0.01%	0.01%	0.01%	0.01%	0.01%
	(0.01%)	(0.01%)	(0.01%)	(0.01%)	(0.00%)
ESG demand risk premium	0.02%	-0.16%	-0.01%	0.16%	0.32%
	(0.00%)	(0.02%)	(0.00%)	(0.02%)	(0.04%)
Convenience yield premium (sample average)	-0.01%	0.14%	0.00%	-0.14%	-0.28%
	(0.01%)	(0.05%)	(0.00%)	(0.05%)	(0.10%)
Convenience yield premium (2018–2020 average)	-0.06%	0.17%	0.03%	-0.16%	-0.33%
	(0.03%)	(0.08%)	(0.02%)	(0.07%)	(0.15%)
Convenience yield premium (2030 projection)	-0.21%	0.53%	0.09%	-0.50%	-1.03%
Expected excess return (sample average)	7.48%	8.30%	7.20%	7.38%	-0.92%
	(0.91%)	(0.79%)	(1.29%)	(0.79%)	(0.22%)
Expected excess return (2018–2020 average)	7.43%	8.33%	7.23%	7.36%	-0.97%
	(0.91%)	(0.80%)	(1.29%)	(0.78%)	(0.27%)
Expected excess return (2030 projection)	7.28%	8.69%	7.28%	7.02%	-1.67%
Expected excess return + δ-shock induced return (sample average)	7.51%	7.83%	7.16%	7.78%	-0.05%
	(0.91%)	(0.78%)	(1.29%)	(0.81%)	(0.33%)
Expected excess return + δ-shock induced return (2018–2020 average)	8.08%	3.58%	7.07%	11.96%	8.39%
	(0.92%)	(0.96%)	(1.29%)	(0.98%)	(1.16%)
Excess return volatility	14.91%	16.18%	16.03%	15.12%	8.00%
	(0.24%)	(0.17%)	(0.37%)	(0.21%)	(0.14%)

Online Appendix to "Dynamic ESG Equilibrium"

Doron Avramov   Abraham Lioui   Yang Liu   Andrea Tarelli

A Proof of Propositions 1: Euler equation

The optimization program is formulated as

$$U_t = \max_{C_t, \boldsymbol{\omega}_t} \left((1-\beta) A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[U_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}, \tag{A.1}$$

$$A_t = C_t + \delta_t G_{W,t} \left(W_t - C_t \right), \tag{A.2}$$

where $G_{W,t} = \sum_{n=1}^N \omega_{n,t} G_{n,t}$ is the aggregate greenness (ESG supply), and δ_t represents time-varying preferences for ESG (demand). The budget constraint states that $W_{t+1} = (W_t - C_t) R_{W,t+1}$, where $R_{W,t+1} = R_{f,t+1} + \sum_{n=1}^N \omega_{n,t} \left(R_{n,t+1} - R_{f,t+1} \right)$. At the optimum, the value function depends on wealth only, that is $U_t = J(W_t)$. The agent then optimizes

$$J(W_t) = \max_{C_t, \boldsymbol{\omega}_t} \left((1-\beta) A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\psi}}}. \tag{A.3}$$

The first order condition with respect to consumption is given by

$$\begin{aligned} 0 =& (1-\beta) A_t^{-\frac{1}{\psi}} (1 - \delta_t G_{W,t}) \\ &- \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} R_{W,t+1} \right], \end{aligned} \tag{A.4}$$

from which we obtain $\mathrm{E}_t \left[M_{t+1} R_{W,t+1} \right] = 1 - \delta_t G_{W,t}$, where M_{t+1}, the stochastic discount factor (SDF), is formulated as

$$M_{t+1} = \beta \frac{\mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}}}{(1-\beta) A_t^{-\frac{1}{\psi}}}. \tag{A.5}$$

Next, we derive the first order condition with respect to $\omega_{n,t}$

$$\begin{aligned} 0 =& \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J(W_{t+1})^{-\gamma} \frac{\partial J(W_{t+1})}{\partial W_{t+1}} (W_t - C_t) (R_{n,t+1} - R_{f,t+1}) \right] \\ &+ (1-\beta) A_t^{-\frac{1}{\psi}} \delta_t G_{n,t} (W_t - C_t). \end{aligned} \tag{A.6}$$

Multiplying (A.6) by $\omega_{n,t}$ and summing up yields

$$\begin{aligned} 0 = &\beta \mathrm{E}_t \left[J\left(W_{t+1}\right)^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J\left(W_{t+1}\right)^{-\gamma} \frac{\partial J\left(W_{t+1}\right)}{\partial W_{t+1}} \left(W_t - C_t\right) R_{W,t+1} \right] \\ &- \beta \mathrm{E}_t \left[J\left(W_{t+1}\right)^{1-\gamma} \right]^{\frac{1}{\theta}-1} \mathrm{E}_t \left[J\left(W_{t+1}\right)^{-\gamma} \frac{\partial J\left(W_{t+1}\right)}{\partial W_{t+1}} \left(W_t - C_t\right) R_{f,t+1} \right] \\ &+ \left(1-\beta\right) A_t^{-\frac{1}{\psi}} \delta_t G_{W,t} \left(W_t - C_t\right). \end{aligned} \tag{A.7}$$

Combining (A.4) and (A.7), we get

$$\mathrm{E}_t \left[\beta \frac{\mathrm{E}_t \left[J\left(W_{t+1}\right)^{1-\gamma} \right]^{\frac{1}{\theta}-1} J\left(W_{t+1}\right)^{-\gamma} \frac{\partial J\left(W_{t+1}\right)}{\partial W_{t+1}}}{\left(1-\beta\right) A_t^{-\frac{1}{\psi}}} R_{f,t+1} \right] = 1, \tag{A.8}$$

which is the Euler equation for the risk-free gross return. Because the risk-free asset is assumed to be, without loss of generality, ESG neutral, the Euler equation for the risk-free rate is written using the standard representation

$$\mathrm{E}_t \left[M_{t+1} R_{f,t+1} \right] = 1. \tag{A.9}$$

From (A.6), we can express the Euler equation for excess return on a generic asset as

$$\mathrm{E}_t \left[M_{t+1} \left(R_{n,t+1} - R_{f,t+1} \right) \right] = -\delta_t G_{n,t}. \tag{A.10}$$

Summing (A.9) and (A.10), we obtain the Euler equation for the gross return on a generic asset:

$$\mathrm{E}_t \left[M_{t+1} R_{n,t+1} \right] = 1 - \delta_t G_{n,t}. \tag{A.11}$$

We next derive an explicit solution for the value function. To start, we guess $J\left(W_t\right) = \Phi_t W_t$. Then, equations (A.3) and (A.4) can be expressed as

$$\beta \mathrm{E}_t \left[\Phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}} = \Phi_t^{1-\frac{1}{\psi}} W_t^{1-\frac{1}{\psi}} - \left(1-\beta\right) A_t^{1-\frac{1}{\psi}} \tag{A.12}$$

and

$$0 = \left(1-\beta\right) A_t^{-\frac{1}{\psi}} \left(1 - \delta_t G_{W,t}\right) \left(W_t - C_t\right) - \beta \mathrm{E}_t \left[\Phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}}, \tag{A.13}$$

respectively. Combining both equations yields $\Phi_t = \left(1-\beta\right)^{\frac{1}{1-\frac{1}{\psi}}} \left(\frac{W_t}{A_t}\right)^{\frac{\frac{1}{\psi}}{1-\frac{1}{\psi}}}$. Then, M_{t+1} in (A.5) can be developed as

$$M_{t+1} = \beta \frac{\mathrm{E}_t \left[\Phi_{t+1}^{1-\gamma} W_{t+1}^{1-\gamma} \right]^{\frac{1}{\theta}-1} \Phi_{t+1}^{1-\gamma} W_{t+1}^{-\gamma}}{\left(1-\beta\right) A_t^{-\frac{1}{\psi}}}$$

$$
\begin{aligned}
&= \beta^{\theta}\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{W_{t+1}}{W_t - A_t}\right)^{\theta-1} \\
&= \beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\psi}}\left(\frac{1+\delta_{t+1}\frac{W_{t+1}-C_{t+1}}{C_{t+1}}G_{W,t+1}}{1+\delta_t\frac{P_t}{C_t}G_{W,t}}\right)^{-\frac{\theta}{\psi}}\left(\frac{W_{t+1}}{W_t - C_t}\frac{1}{1-\delta_t G_{W,t}}\right)^{\theta-1}.
\end{aligned}
\tag{A.14}
$$

Finally, the Euler equation (A.11) and the corresponding SDF can be expressed as

$$
\mathrm{E}_t\left[M_{t+1}\tilde{R}_{n,t+1}\right] = 1, \tag{A.15}
$$

$$
M_{t+1} = \beta^{\theta}\left(\frac{A_{t+1}}{A_t}\right)^{-\frac{\theta}{\psi}}\tilde{R}_{W,t+1}^{\theta-1}, \tag{A.16}
$$

where $\tilde{R}_{W,t+1} = \frac{R_{W,t+1}}{1-\delta_t G_{W,t}}$ and $\tilde{R}_{n,t+1} = \frac{R_{n,t+1}}{1-\delta_t G_{W,t}}$ are the ESG-adjusted gross returns on the consumption asset and on a generic asset, respectively. The Euler equation undertakes the standard form only when the financial return is replaced by ESG adjusted return.

In what follows, we resort to the log of the SDF, $m_{t+1} = \log M_{t+1}$, which obtains as

$$
\begin{aligned}
m_{t+1} = {} & \theta\log\beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)\left(r_{W,t+1} - \log\left(1-\delta_t G_{W,t}\right)\right) \\
& - \frac{\theta}{\psi}\log\left(\frac{1+\delta_{t+1}\frac{P_{t+1}}{C_{t+1}}G_{W,t+1}}{1+\delta_t\frac{P_t}{C_t}G_{W,t}}\right),
\end{aligned}
\tag{A.17}
$$

where $\Delta c_{t+1} = \log\frac{C_{t+1}}{C_t}$ and $r_{W,t+1} = \log\frac{W_{t+1}}{W_t - C_t}$ is the logarithmic return on financial wealth. The expected excess return of a generic asset then satisfies the following relation

$$
\mathrm{E}_t\left[r_{n,t+1} - r_{f,t+1}\right] + \frac{1}{2}\mathrm{Var}_t\left[r_{n,t+1}\right] = -\mathrm{Cov}_t\left[m_{t+1}, r_{n,t+1}\right] - y_{n,t}, \tag{A.18}
$$

where $r_{n,t+1} = \log R_{n,t+1}$, $r_{f,t+1} = \log R_{f,t+1}$, and $y_{n,t} = -\log\left(1-\delta_t G_{n,t}\right)$. Equation (6) is obtained combining (A.17) and (A.18).

Finally, we aim to determine the concavity of the value function with respect to $G_{W,t}$ and δ_t. We start evaluating the first derivatives:

$$
\begin{aligned}
\frac{\partial J(W_t)}{\partial G_{W,t}} &= (1-\beta)\left((1-\beta)A_t^{1-\frac{1}{\psi}} + \beta\mathrm{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}-1} A_t^{-\frac{1}{\psi}}\delta_t\left(W_t - C_t\right) \\
&= (1-\beta)J(W_t)^{\frac{1}{\psi}}A_t^{-\frac{1}{\psi}}\delta_t\left(W_t - C_t\right),
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
\frac{\partial J(W_t)}{\partial \delta_t} &= (1-\beta)\left((1-\beta)A_t^{1-\frac{1}{\psi}} + \beta\mathrm{E}_t\left[U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right)^{\frac{1}{1-\frac{1}{\psi}}-1} A_t^{-\frac{1}{\psi}}G_{W,t}\left(W_t - C_t\right) \\
&= (1-\beta)J(W_t)^{\frac{1}{\psi}}A_t^{-\frac{1}{\psi}}G_{W,t}\left(W_t - C_t\right),
\end{aligned}
\tag{A.20}
$$

which, respectively, are positive for $\delta_t > 0$ and $G_{W,t} > 0$. The second derivatives are

$$\frac{\partial^2 J(W_t)}{\partial G_{W,t}^2} = -\frac{1}{\psi}(1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}-1} \delta_t^2 (W_t - C_t)^2$$
$$\cdot \frac{\beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}}{(1-\beta) A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}} < 0, \tag{A.21}$$

$$\frac{\partial^2 J(W_t)}{\partial \delta_t^2} = -\frac{1}{\psi}(1-\beta) J(W_t)^{\frac{1}{\psi}} A_t^{-\frac{1}{\psi}-1} G_{W,t}^2 (W_t - C_t)^2$$
$$\cdot \frac{\beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}}{(1-\beta) A_t^{1-\frac{1}{\psi}} + \beta \mathrm{E}_t \left[J(W_{t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}} < 0, \tag{A.22}$$

which are both negative. The value function is thus concave in $G_{W,t}$ and δ_t.

B Proof of Proposition 2: Stochastic discount factor

We consider first the standard log-linearization

$$r_{W,t+1} \simeq \kappa_{rW,0} + \kappa_{rW,pc} pc_{t+1} - pc_t + \Delta c_{t+1}, \tag{A.23}$$

where pc_t is the log price/consumption ratio, $\kappa_{rW,pc} = \frac{e^{\overline{pc}}}{1+e^{\overline{pc}}}$, $\kappa_{rW,0} = \log\left(1+e^{\overline{pc}}\right) - \kappa_{rW,pc}\overline{pc}$, with $\overline{pc}$, the model-implied average log price/consumption ratio, being determined as the solution of a fixed-point problem. We perform two additional approximations. The first is given by

$$\log\left(1 + e^{pc_t}\delta_t G_{W,t}\right) \simeq \log\left(1 + e^{\overline{pc}}\bar{\delta}\bar{G}_W\right) + \frac{e^{\overline{pc}}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(\delta_t - \bar{\delta}\right)$$
$$+ \frac{e^{\overline{pc}}\bar{\delta}}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(G_{W,t} - \bar{G}_W\right) + \frac{e^{\overline{pc}}\bar{\delta}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}\left(pc_t - \overline{pc}\right)$$
$$= \kappa_{m,0} + \kappa_{m,\delta}\delta_t + \kappa_{m,G}G_{W,t} + \kappa_{m,pc}pc_t, \tag{A.24}$$

where $\kappa_{m,\delta} = \frac{e^{\overline{pc}}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, $\kappa_{m,G} = \frac{e^{\overline{pc}}\bar{\delta}}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, $\kappa_{m,pc} = \frac{e^{\overline{pc}}\bar{\delta}\bar{G}_W}{1+e^{\overline{pc}}\bar{\delta}\bar{G}_W}$, and $\kappa_{m,0} = \log\left(1+e^{\overline{pc}}\bar{\delta}\bar{G}_W\right) - \kappa_{m,\delta}\bar{\delta} - \kappa_{m,G}\bar{G}_W - \kappa_{m,pc}\overline{pc}$. The second approximation is

$$\log\left(1 - \delta_t G_{W,t}\right) \simeq \log\left(1 - \bar{\delta}\bar{G}_W\right) - \frac{\bar{G}_W}{1-\bar{\delta}\bar{G}_W}\left(\delta_t - \bar{\delta}\right) - \frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_W}\left(G_{W,t} - \bar{G}_W\right)$$
$$= \kappa_{W,0} + \kappa_{W,\delta}\delta_t + \kappa_{W,G}G_{W,t}, \tag{A.25}$$

where $\kappa_{W,\delta} = -\frac{\bar{G}_W}{1-\bar{\delta}\bar{G}_W}$, $\kappa_{W,G} = -\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_W}$, and $\kappa_{W,0} = \log\left(1-\bar{\delta}\bar{G}_W\right) - \kappa_{W,\delta}\bar{\delta} - \kappa_{W,G}\bar{G}_W$. Then, we can rewrite the SDF as

$$\begin{aligned} m_{t+1} \simeq \theta \log \beta - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta-1)\, r_{W,t+1} - (\theta-1)\left(\kappa_{W,0} + \kappa_{W,\delta}\delta_t + \kappa_{W,G}G_{W,t}\right) \\ - \frac{\theta}{\psi}\kappa_{m,\delta}\Delta\delta_{t+1} - \frac{\theta}{\psi}\kappa_{m,G}\Delta G_{W,t+1} - \frac{\theta}{\psi}\kappa_{m,pc}\Delta pc_{t+1}. \end{aligned} \tag{A.26}$$

From the theory section, we specify four dynamic processes:

$$\Delta c_{t+1} = \mu_c + x_t + \sigma_c \varepsilon_{c,t+1}, \tag{A.27}$$

$$G_{W,t+1} = \mu_G + \rho_G G_{W,t} + \sigma_G \varepsilon_{G,t+1}, \tag{A.28}$$

$$\delta_{t+1} = \mu_\delta + \rho_\delta \delta_t + \sigma_\delta \varepsilon_{\delta,t+1}, \tag{A.29}$$

$$x_{t+1} = \rho_x x_t + \sigma_x \varepsilon_{x,t+1}, \tag{A.30}$$

where $G_{W,t+1}$ and δ_{t+1} are mean reverting, $\mu_G = (1-\rho_G)\,\bar{G}_W$, and $\mu_\delta = (1-\rho_\delta)\,\bar{\delta}$. Based on (A.25), we rewrite the Euler equation as:

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{W,t+1}}\right] = e^{\kappa_{W,0}+\kappa_{W,\delta}\delta_t+\kappa_{W,G}G_{W,t}}. \tag{A.31}$$

To characterize the SDF, we make the guess:

$$pc_t = A_{pc,0} + A_{pc,G}G_{W,t} + A_{pc,\delta}\delta_t + A_{pc,x}x_t. \tag{A.32}$$

Then, it follows that

$$\begin{aligned} r_{W,t+1} &\simeq \kappa_{rW,0} + \kappa_{rW,pc}pc_{t+1} - pc_t + \Delta c_{t+1} \\ &= \kappa_{rW,0} + A_{pc,0}\left(\kappa_{rW,pc}-1\right) + A_{pc,G}\left(\kappa_{rW,pc}G_{W,t+1} - G_{W,t}\right) \\ &\quad + A_{pc,\delta}\left(\kappa_{rW,pc}\delta_{t+1} - \delta_t\right) + A_{pc,x}\left(\kappa_{rW,pc}x_{t+1} - x_t\right) + \Delta c_{t+1}. \end{aligned} \tag{A.33}$$

Following tedious algebra, we obtain

$$\begin{aligned} &m_{t+1} + r_{W,t+1} = \\ &\quad \theta\log\beta - (\theta-1)\,\kappa_{W,0} + (1-\gamma)\,\mu_c + \theta\kappa_{rW,0} + \theta A_{pc,0}\left(\kappa_{rW,pc}-1\right) \\ &\quad + \theta\kappa_{rW,pc}\left(A_{pc,G}\mu_G + A_{pc,\delta}\mu_\delta\right) \\ &\quad - \frac{\theta}{\psi}\left(\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\mu_G + \left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\mu_\delta\right) \\ &\quad + \left(\theta A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1 - \frac{\kappa_{m,pc}}{\psi}\left(\rho_G-1\right)\right) - (\theta-1)\,\kappa_{W,G} - \frac{\theta}{\psi}\kappa_{m,G}\left(\rho_G-1\right)\right) G_{W,t} \\ &\quad + \left(\theta A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1 - \frac{\kappa_{m,pc}}{\psi}\left(\rho_\delta-1\right)\right) - (\theta-1)\,\kappa_{W,\delta} - \frac{\theta}{\psi}\kappa_{m,\delta}\left(\rho_\delta-1\right)\right)\delta_t \end{aligned}$$

$$+\left((1-\gamma)+\theta A_{pc,x}\left(\kappa_{rW,pc}\rho_x-1-\frac{\kappa_{m,pc}}{\psi}(\rho_x-1)\right)\right)x_t$$
$$+(1-\gamma)\sigma_c\varepsilon_{c,t+1}$$
$$+\left(\theta A_{pc,G}\left(\kappa_{rW,pc}-\frac{\kappa_{m,pc}}{\psi}\right)-\frac{\theta}{\psi}\kappa_{m,G}\right)\sigma_G\varepsilon_{G,t+1}$$
$$+\left(\theta A_{pc,\delta}\left(\kappa_{rW,pc}-\frac{\kappa_{m,pc}}{\psi}\right)-\frac{\theta}{\psi}\kappa_{m,\delta}\right)\sigma_\delta\varepsilon_{\delta,t+1}$$
$$+\theta A_{pc,x}\left(\kappa_{rW,pc}-\frac{\kappa_{m,pc}}{\psi}\right)\sigma_x\varepsilon_{x,t+1}. \tag{A.34}$$

As $\mathrm{E}_t\left[e^{m_{t+1}+r_{W,t+1}}\right]=e^{\kappa_{W,0}+\kappa_{W,\delta}\delta_t+\kappa_{W,G}G_{W,t}}$, we can solve for the coefficients:

$$A_{pc,0}=\frac{1}{\theta(1-\kappa_{rW,pc})}\begin{pmatrix}\theta\log\beta-\theta\kappa_{W,0}+(1-\gamma)\mu_c\\ +\theta\kappa_{rW,0}+\theta\kappa_{rW,pc}(A_{pc,G}\mu_G+A_{pc,\delta}\mu_\delta)\\ -\frac{\theta}{\psi}(\kappa_{m,G}+\kappa_{m,pc}A_{pc,G})\mu_G\\ -\frac{\theta}{\psi}(\kappa_{m,\delta}+\kappa_{m,pc}A_{pc,\delta})\mu_\delta\\ +\frac{(1-\gamma)^2\sigma_c^2}{2}+\frac{\left(\theta A_{pc,G}\left(\kappa_{rW,pc}-\frac{\kappa_{m,pc}}{\psi}\right)-\frac{\theta}{\psi}\kappa_{m,G}\right)^2\sigma_G^2}{2}\\ +\frac{\left(\theta A_{pc,\delta}\left(\kappa_{rW,pc}-\frac{\kappa_{m,pc}}{\psi}\right)-\frac{\theta}{\psi}\kappa_{m,\delta}\right)^2\sigma_\delta^2}{2}\\ +\frac{\left(\theta\kappa_{rW,pc}-\frac{\theta}{\psi}\kappa_{m,pc}\right)^2A_{pc,x}^2\sigma_x^2}{2}\end{pmatrix}, \tag{A.35}$$

$$A_{pc,G}=\frac{\kappa_{m,G}(1-\rho_G)-\psi\kappa_{W,G}}{\psi-\kappa_{m,pc}-(\psi\kappa_{rW,pc}-\kappa_{m,pc})\rho_G}=\frac{\kappa_{m,G}-\frac{\psi}{1-\rho_G}\kappa_{W,G}}{\psi\frac{1-\kappa_{rW,pc}\rho_G}{1-\rho_G}-\kappa_{m,pc}}, \tag{A.36}$$

$$A_{pc,\delta}=\frac{\kappa_{m,\delta}(1-\rho_\delta)-\psi\kappa_{W,\delta}}{\psi-\kappa_{m,pc}-(\psi\kappa_{rW,pc}-\kappa_{m,pc})\rho_\delta}=\frac{\kappa_{m,\delta}-\frac{\psi}{1-\rho_\delta}\kappa_{W,\delta}}{\psi\frac{1-\kappa_{rW,pc}\rho_\delta}{1-\rho_\delta}-\kappa_{m,pc}}, \tag{A.37}$$

$$A_{pc,x}=\frac{\psi-1}{\psi-\kappa_{m,pc}-(\psi\kappa_{rW,pc}-\kappa_{m,pc})\rho_x}=\frac{\frac{\psi-1}{1-\rho_x}}{\psi\frac{1-\kappa_{rW,pc}\rho_x}{1-\rho_x}-\kappa_{m,pc}}. \tag{A.38}$$

A sufficient condition for the coefficients $A_{pc,G}$, $A_{pc,\delta}$, and $A_{pc,x}$ to be positive is that $\psi>1$, $\bar{\delta}\geq 0$, and $\bar{G}_W\geq 0$. We can identify the market prices of risk rewriting m_{t+1} as:

$$m_{t+1}=m_0+m_GG_{W,t}+m_\delta\delta_t+m_xx_t-\lambda_c\varepsilon_{c,t+1}-\lambda_G\varepsilon_{G,t+1}-\lambda_\delta\varepsilon_{\delta,t+1}-\lambda_x\varepsilon_{x,t+1}, \tag{A.39}$$

where

$$m_0=\theta\log\beta-\gamma\mu_c+(\theta-1)(\kappa_{rW,0}-\kappa_{W,0}+A_{pc,0}(\kappa_{rW,pc}-1))$$
$$+(\theta-1)\kappa_{rW,pc}(A_{pc,G}\mu_G+A_{pc,\delta}\mu_\delta) \tag{A.40}$$
$$-\frac{\theta}{\psi}((\kappa_{m,\delta}+\kappa_{m,pc}A_{pc,\delta})\mu_\delta+(\kappa_{m,G}+\kappa_{m,pc}A_{pc,G})\mu_G), \tag{A.41}$$
$$m_G=(\theta-1)(A_{pc,G}(\kappa_{rW,pc}\rho_G-1)-\kappa_{W,G})-\frac{\theta}{\psi}(\kappa_{m,G}+\kappa_{m,pc}A_{pc,G})(\rho_G-1)$$

$$= \frac{\frac{\kappa_{m,G}}{\psi}\left(1-\rho_G\right) - \kappa_{W,G}\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_G}{1-\kappa_{rW,pc}\rho_G}}{1-\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_G}{1-\kappa_{rW,pc}\rho_G}}, \tag{A.42}$$

$$m_\delta = (\theta-1)\left(A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1\right) - \kappa_{W,\delta}\right) - \frac{\theta}{\psi}\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\left(\rho_\delta - 1\right)$$

$$= \frac{\frac{\kappa_{m,\delta}}{\psi}\left(1-\rho_\delta\right) - \kappa_{W,\delta}\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_\delta}{1-\kappa_{rW,pc}\rho_\delta}}{1-\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_\delta}{1-\kappa_{rW,pc}\rho_\delta}}, \tag{A.43}$$

$$m_x = -\gamma + \left((\theta-1)\left(\kappa_{rW,pc}\rho_x - 1\right) - \frac{\theta}{\psi}\kappa_{m,pc}\left(\rho_x - 1\right)\right)A_{pc,x}$$

$$= -\frac{1}{\psi}\frac{1-\kappa_{m,pc}\frac{1-\rho_x}{1-\kappa_{rW,pc}\rho_x}}{1-\frac{\kappa_{m,pc}}{\psi}\frac{1-\rho_x}{1-\kappa_{rW,pc}\rho_x}}, \tag{A.44}$$

$$\lambda_c = \gamma\sigma_c, \tag{A.45}$$

$$\lambda_G = \left((1-\theta)\,\kappa_{rW,pc}A_{pc,G} + \frac{\theta}{\psi}\left(\kappa_{m,G} + \kappa_{m,pc}A_{pc,G}\right)\right)\sigma_G, \tag{A.46}$$

$$\lambda_\delta = \left((1-\theta)\,\kappa_{rW,pc}A_{pc,\delta} + \frac{\theta}{\psi}\left(\kappa_{m,\delta} + \kappa_{m,pc}A_{pc,\delta}\right)\right)\sigma_\delta, \tag{A.47}$$

$$\lambda_x = \left((1-\theta)\,\kappa_{rW,pc} + \frac{\theta}{\psi}\kappa_{m,pc}\right)A_{pc,x}\sigma_x \tag{A.48}$$

$$= \left(\frac{\gamma\psi-1}{\psi-1}\kappa_{rW,pc} - \frac{\gamma-1}{\psi-1}\kappa_{m,pc}\right)A_{pc,x}\sigma_x. \tag{A.49}$$

Assuming that $\left|\bar{\delta}\bar{G}_W\right| < 1$, it follows that $m_x < 0$. Assuming that $\psi > 1$, $\bar{\delta} \geq 0$, and $\bar{G}_W \geq 0$ is a sufficient condition for $m_G, m_\delta > 0$, as well as $-1 < m_x < 0$. As for the market prices of risk, it is useful to notice that $\kappa_{rW,pc} > \kappa_{m,pc}$. It turns out that $\lambda_c, \lambda_x > 0$ at all times. The signs of λ_G and λ_δ depend on both positive and negative contributions. For instance, for $\theta < 0$, they are characterized by positive contributions stemming from the impact of shocks to $G_{W,t}$ and δ_t on the return on aggregate wealth, while negative contributions arise from the effect on the ESG factor.

The return on wealth can be formulated as

$$\begin{aligned} r_{W,t+1} = {} & \underbrace{\kappa_{rW,0} + A_{pc,0}\left(\kappa_{rW,pc} - 1\right) + A_{pc,G}\kappa_{rW,pc}\mu_G + A_{pc,\delta}\kappa_{rW,pc}\mu_\delta + \mu_c}_{r_{W,0}} \\ & + A_{pc,G}\left(\kappa_{rW,pc}\rho_G - 1\right)G_{W,t} + A_{pc,\delta}\left(\kappa_{rW,pc}\rho_\delta - 1\right)\delta_t \\ & + \left(A_{pc,x}\left(\kappa_{rW,pc}\rho_x - 1\right) + 1\right)x_t \\ & + A_{pc,G}\kappa_{rW,pc}\sigma_G\varepsilon_{G,t+1} + A_{pc,\delta}\kappa_{rW,pc}\sigma_\delta\varepsilon_{\delta,t+1} \\ & + A_{pc,x}\kappa_{rW,pc}\sigma_x\varepsilon_{x,t+1} + \sigma_c\varepsilon_{c,t+1}. \end{aligned} \tag{A.50}$$

As $A_{pc,G}$, $A_{pc,\delta}$, and $A_{pc,x}$ are positive, the return on wealth is positively correlated with the shocks $\varepsilon_{G,t+1}$, $\varepsilon_{\delta,t+1}$, $\varepsilon_{x,t+1}$, and $\varepsilon_{c,t+1}$. The expected excess return of the consumption asset can be expressed as

$$\begin{aligned}\mathrm{E}_t\left[r_{W,t+1}-r_{f,t+1}\right]+\frac{1}{2}\mathrm{Var}_t\left[r_{W,t+1}\right] = & \underbrace{\sigma_c}_{\beta_{M,c}\sigma_c^2}\lambda_c+\underbrace{\kappa_{rW,pc}A_{pc,G}\sigma_G}_{\beta_{M,G}\sigma_G^2}\lambda_G \\ & +\underbrace{\kappa_{rW,pc}A_{pc,\delta}\sigma_\delta}_{\beta_{M,\delta}\sigma_\delta^2}\lambda_\delta+\underbrace{\kappa_{rW,pc}A_{pc,x}\sigma_x}_{\beta_{M,x}\sigma_x^2}\lambda_x \\ & +\underbrace{\kappa_{W,0}-\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_W}G_{W,t}-\frac{\bar{G}_W}{1-\bar{\delta}\bar{G}_W}\delta_t}_{-y_{M,t}}. \end{aligned} \quad (A.51)$$

C Proof of Proposition 3: Risk-free return

We express the Euler equation as

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{f,t+1}}\right]=1. \quad (A.52)$$

As $r_{f,t+1}$ is known at time t, it follows that $\mathrm{E}_t\left[e^{r_{f,t+1}}\right]=e^{\mathrm{E}_t\left[r_{f,t+1}\right]}$. Thus, the risk-free rate of return is given by

$$r_{f,t+1}=-\log\mathrm{E}_t\left[e^{m_{t+1}}\right]. \quad (A.53)$$

Using (A.39):

$$\begin{aligned}\mathrm{E}_t\left[e^{m_{t+1}}\right] &= \mathrm{E}_t\left[e^{m_0+m_G G_{W,t}+m_\delta\delta_t+m_x x_t-\lambda_c\varepsilon_{c,t+1}-\lambda_G\varepsilon_{G,t+1}-\lambda_\delta\varepsilon_{\delta,t+1}-\lambda_x\varepsilon_{x,t+1}}\right] \\ &= e^{m_0+m_G G_{W,t}+m_\delta\delta_t+m_x x_t+\frac{\lambda_c^2}{2}+\frac{\lambda_G^2}{2}+\frac{\lambda_\delta^2}{2}+\frac{\lambda_x^2}{2}}. \end{aligned} \quad (A.54)$$

The, using (A.32) yields

$$r_{f,t+1}=\underbrace{-m_0-\frac{\lambda_c^2}{2}-\frac{\lambda_G^2}{2}-\frac{\lambda_\delta^2}{2}-\frac{\lambda_x^2}{2}}_{r_{f,0}}\underbrace{-m_G}_{r_{f,G}}G_{W,t}\underbrace{-m_\delta}_{r_{f,\delta}}\delta_t\underbrace{-m_x}_{r_{f,x}}x_t. \quad (A.55)$$

D Proof of Proposition 4: Market return

For the market portfolio, we assume $G_{M,t}=G_{W,t}$ and thus we can rewrite the Euler condition (A.11) as

$$\mathrm{E}_t\left[M_{t+1}R_{M,t+1}\right]=1-\delta_t G_{W,t}. \quad (A.56)$$

Recalling (A.25), we can write $\log\left(1-\delta_t G_{W,t}\right)\simeq\kappa_{W,0}+\kappa_{W,G}G_{W,t}+\kappa_{W,\delta}\delta_t$.

We use the following log-linearization for the return of the market portfolio:

$$r_{M,t+1}\simeq\kappa_{rM,0}+\kappa_{rM,pd}pd_{M,t+1}-pd_{M,t}+\Delta d_{M,t+1}, \quad (A.57)$$

where $\kappa_{rM,pd} = \frac{e^{\overline{pd}_M}}{1+e^{\overline{pd}_M}}$ and $\kappa_{rM,0} = \log\left(1 + e^{\overline{pd}_M}\right) - \kappa_{rM,pd}\overline{pd}_M$. Consider the following dynamics:

$$\begin{aligned}\Delta d_{M,t+1} =& \mu_{dM} + \rho_{dM,x}x_t + \rho_{dM,\delta}\delta_t \\ &+ \sigma_{dM,c}\varepsilon_{c,t+1} + \sigma_{dM,G}\varepsilon_{G,t+1} + \sigma_{dM,\delta}\varepsilon_{\delta,t+1} + \sigma_{dM,x}\varepsilon_{x,t+1} + \sigma_{dM}\varepsilon_{dM,t+1}.\end{aligned} \tag{A.58}$$

We make the guess:

$$pd_{M,t} = A_{M,0} + A_{M,G}G_{W,t} + A_{M,\delta}\delta_t + A_{M,x}x_t. \tag{A.59}$$

We then write the log market return as:

$$\begin{aligned}r_{M,t+1} \simeq& \underbrace{\kappa_{rM,0} + \kappa_{rM,pd}\left(A_{M,0} + A_{M,G}\mu_G + A_{M,\delta}\mu_\delta\right) - A_{M,0} + \mu_{dM}}_{r_{M,0}} \\ &+ \underbrace{\left(\kappa_{rM,pd}\rho_G - 1\right)A_{M,G}}_{r_{M,G}} G_{W,t} + \underbrace{\left(\left(\kappa_{rM,pd}\rho_\delta - 1\right)A_{M,\delta} + \rho_{dM,\delta}\right)}_{r_{M,\delta}}\delta_t \\ &+ \underbrace{\left(\left(\kappa_{rM,pd}\rho_x - 1\right)A_{M,x} + \rho_{dM,x}\right)}_{r_{M,x}}x_t \\ &+ \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}}\varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}}\varepsilon_{G,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}}\varepsilon_{\delta,t+1} \\ &+ \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}}\varepsilon_{x,t+1} + \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}}\varepsilon_{dM,t+1}.\end{aligned} \tag{A.60}$$

We impose the Euler condition

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{M,t+1}}\right] \simeq e^{\kappa_{W,0}+\kappa_{W,G}G_{W,t}+\kappa_{W,\delta}\delta_t}, \tag{A.61}$$

where

$$\begin{aligned}r_{M,t+1} + m_{t+1} \simeq& \kappa_{rM,0} + \kappa_{rM,pd}\left(A_{M,0} + A_{M,G}\mu_G + A_{M,\delta}\mu_\delta\right) - A_{M,0} + \mu_{dM} + m_0 \\ &+ \left(\left(\kappa_{rM,pd}\rho_G - 1\right)A_{M,G} + m_G\right)G_{W,t} \\ &+ \left(\left(\kappa_{rM,pd}\rho_\delta - 1\right)A_{M,\delta} + \rho_{dM,\delta} + m_\delta\right)\delta_t \\ &+ \left(\left(\kappa_{rM,pd}\rho_x - 1\right)A_{M,x} + \rho_{dM,x} + m_x\right)x_t \\ &+ \left(\sigma_{dM,c} - \lambda_c\right)\varepsilon_{c,t+1} + \left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G} - \lambda_G\right)\varepsilon_{G,t+1} \\ &+ \left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta} - \lambda_\delta\right)\varepsilon_{\delta,t+1} \\ &+ \left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x} - \lambda_x\right)\varepsilon_{x,t+1} + \sigma_{dM}\varepsilon_{dM,t+1}.\end{aligned} \tag{A.62}$$

Therefore

$$
\begin{aligned}
0 \simeq\, & \kappa_{rM,0} + \kappa_{rM,pd}\left(A_{M,G}\mu_G + A_{M,\delta}\mu_\delta\right) + \left(\kappa_{rM,pd} - 1\right) A_{M,0} + \mu_{dM} + m_0 \\
& - \kappa_{W,0} + \frac{\left(\sigma_{dM,c} - \lambda_c\right)^2}{2} + \frac{\left(\kappa_{rM,pd} A_{M,G}\sigma_G + \sigma_{dM,G} - \lambda_G\right)^2}{2} \\
& + \frac{\left(\kappa_{rM,pd} A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta} - \lambda_\delta\right)^2}{2} + \frac{\left(\kappa_{rM,pd} A_{M,x}\sigma_x + \sigma_{dM,x} - \lambda_x\right)^2}{2} + \frac{\sigma_{dM}^2}{2} \\
& + \left(\left(\kappa_{rM,pd}\rho_G - 1\right) A_{M,G} + m_G - \kappa_{W,G}\right) G_{W,t} \\
& + \left(\left(\kappa_{rM,pd}\rho_\delta - 1\right) A_{M,\delta} + \rho_{dM,\delta} + m_\delta - \kappa_{W,\delta}\right) \delta_t \\
& + \left(\left(\kappa_{rM,pd}\rho_x - 1\right) A_{M,x} + \rho_{dM,x} + m_x\right) x_t.
\end{aligned} \tag{A.63}
$$

Finally, the coefficients in (A.59) are

$$
A_{M,0} = \frac{1}{1-\kappa_{rM,pd}} \left(\begin{array}{c} \kappa_{rM,0} + \kappa_{rM,pd}\left(A_{M,G}\mu_G + A_{M,\delta}\mu_\delta\right) + \mu_{dM} + m_0 \\ -\kappa_{W,0} + \frac{\sigma_{dM}^2}{2} + \frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2} + \frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ +\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} + \frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} \end{array} \right), \tag{A.64}
$$

$$
A_{M,G} = \frac{m_G - \kappa_{W,G}}{1 - \kappa_{rM,pd}\rho_G}, \tag{A.65}
$$

$$
A_{M,\delta} = \frac{m_\delta + \rho_{dM,\delta} - \kappa_{W,\delta}}{1 - \kappa_{rM,pd}\rho_\delta}, \tag{A.66}
$$

$$
A_{M,x} = \frac{m_x + \rho_{dM,x}}{1 - \kappa_{rM,pd}\rho_x}. \tag{A.67}
$$

The return can be then rewritten as:

$$
\begin{aligned}
r_{M,t+1} \simeq\, & \underbrace{\left(\begin{array}{c} -m_0 + \kappa_{W,0} - \frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ -\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} - \frac{\sigma_{dM}^2}{2} \end{array} \right)}_{r_{M,0}} \\
& + \underbrace{\left(\kappa_{W,G} - m_G\right)}_{r_{M,G}} G_{W,t} + \underbrace{\left(\kappa_{W,\delta} - m_\delta\right)}_{r_{M,\delta}} \delta_t \underbrace{-m_x}_{r_{M,x}} x_t \\
& + \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}} \varepsilon_{G,t+1} \\
& + \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}} \varepsilon_{\delta,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}} \varepsilon_{x,t+1} \\
& + \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}} \varepsilon_{dM,t+1}.
\end{aligned} \tag{A.68}
$$

Recalling (15), the excess return $\hat{r}_{M,t+1} = r_{M,t+1} - r_{f,t+1}$ is thus

$$\hat{r}_{M,t+1} \simeq \underbrace{\begin{pmatrix} \kappa_{W,0} - \frac{\left(\sigma_{dM,c}-\lambda_c\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,G}\sigma_G+\sigma_{dM,G}-\lambda_G\right)^2}{2} \\ -\frac{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta+\sigma_{dM,\delta}-\lambda_\delta\right)^2}{2} - \frac{\left(\kappa_{rM,pd}A_{M,x}\sigma_x+\sigma_{dM,x}-\lambda_x\right)^2}{2} - \frac{\sigma_{dM}^2}{2} \\ +\frac{\lambda_c^2}{2} + \frac{\lambda_G^2}{2} + \frac{\lambda_\delta^2}{2} + \frac{\lambda_x^2}{2} \end{pmatrix}}_{\hat{r}_{M,0}}$$

$$+ \underbrace{\kappa_{W,G}}_{\hat{r}_{M,G}} G_{W,t} + \underbrace{\kappa_{W,\delta}}_{\hat{r}_{M,\delta}} \delta_t$$

$$+ \underbrace{\sigma_{dM,c}}_{\sigma_{rM,c}} \varepsilon_{c,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,G}\sigma_G + \sigma_{dM,G}\right)}_{\sigma_{rM,G}} \varepsilon_{G,t+1}$$

$$+ \underbrace{\left(\kappa_{rM,pd}A_{M,\delta}\sigma_\delta + \sigma_{dM,\delta}\right)}_{\sigma_{rM,\delta}} \varepsilon_{\delta,t+1} + \underbrace{\left(\kappa_{rM,pd}A_{M,x}\sigma_x + \sigma_{dM,x}\right)}_{\sigma_{rM,x}} \varepsilon_{x,t+1}$$

$$+ \underbrace{\sigma_{dM}}_{\sigma_{rM,dM}} \varepsilon_{dM,t+1}. \tag{A.69}$$

$\psi > 1$ is a sufficient condition for $A_{M,G} > 0$. If in addition $\bar{G}_W > -\frac{m_\delta+\rho_{dM,\delta}}{1-\bar{\delta}\left(m_\delta+\rho_{dM,\delta}\right)}$, then $A_{M,\delta} > 0$ ($\bar{G}_W > 0$ is a sufficient condition for the positivity of $A_{M,\delta}$). In this case, expected returns are negatively correlated with G_t and δ_t, as $r_{M,G}, r_{M,\delta} < 0$. Proposition 4 is obtained imposing $\rho_{dM,\delta} = \sigma_{dM,G} = \sigma_{dM,\delta} = \sigma_{dM,x} = 0$.

E Proof of Proposition 5: Risky asset returns

For an arbitrary risky asset, the Euler equation reads:

$$\mathrm{E}_t\left[M_{t+1}R_{n,t+1}\right] = 1 - \delta_t G_{n,t}. \tag{A.70}$$

Similarly to (A.25), we can write

$$\log\left(1 - \delta_t G_{n,t}\right) \simeq \kappa_{n,0} + \kappa_{n,Gn}G_{n,t} + \kappa_{n,\delta}\delta_t, \tag{A.71}$$

where $\kappa_{n,Gn} = -\frac{\bar{\delta}}{1-\bar{\delta}\bar{G}_n}$, $\kappa_{n,\delta} = -\frac{\bar{G}_n}{1-\bar{\delta}\bar{G}_n}$, and $\kappa_{n,0} = \log\left(1 - \bar{\delta}\bar{G}_n\right) - \kappa_{n,Gn}\bar{G}_n - \kappa_{n,\delta}\bar{\delta}$.

We use the following log-linearization for the return of the risky asset:

$$r_{n,t+1} \simeq \kappa_{rn,0} + \kappa_{rn,pd}pd_{n,t+1} - pd_{n,t} + \Delta d_{n,t+1}, \tag{A.72}$$

where $\kappa_{rn,pd} = \frac{e^{\overline{pd}_n}}{1+e^{\overline{pd}_n}}$ and $\kappa_{rn,0} = \log\left(1 + e^{\overline{pd}_n}\right) - \kappa_{rn,pd}\overline{pd}_n$. Consider the following dynamics:

$$G_{n,t+1} = \mu_{Gn} + \rho_{Gn}G_{n,t} + \sigma_{Gn,G}\varepsilon_{G,t+1} + \sigma_{Gn}\varepsilon_{Gn,t+1}, \tag{A.73}$$

$$\Delta d_{n,t+1} = \mu_{dn} + \rho_{dn,x} x_t + \rho_{dn,\delta}\delta_t + \sigma_{dn,c}\varepsilon_{c,t+1} + \sigma_{dn,G}\varepsilon_{G,t+1} + \sigma_{dn,\delta}\varepsilon_{\delta,t+1} + \sigma_{dn,x}\varepsilon_{x,t+1} + \sigma_{dn,Gn}\varepsilon_{Gn,t+1} + \sigma_{dn,dM}\varepsilon_{dM,t+1} + \sigma_{dn}\varepsilon_{dn,t+1}, \tag{A.74}$$

where $\mu_{Gn} = (1-\rho_{Gn})\bar{G}_n$. We make the guess:

$$pd_{n,t} = A_{n,0} + A_{n,G} G_{W,t} + A_{n,\delta}\delta_t + A_{n,x} x_t + A_{n,Gn} G_{n,t}. \tag{A.75}$$

We then write the log asset return as:

$$\begin{aligned} r_{n,t+1} \simeq\; & \underbrace{\kappa_{rn,0} + \kappa_{rn,pd} A_{n,0} + \kappa_{rn,pd} A_{n,G}\mu_G + \kappa_{rn,pd} A_{n,\delta}\mu_\delta + \kappa_{rn,pd} A_{n,Gn}\mu_{Gn} - A_{n,0} + \mu_{dn}}_{r_{n,0}} \\ & + \underbrace{(\kappa_{rn,pd}\rho_G - 1) A_{n,G}}_{r_{n,G}} G_{W,t} + \underbrace{((\kappa_{rn,pd}\rho_\delta - 1) A_{n,\delta} + \rho_{dn,\delta})}_{r_{n,\delta}} \delta_t \\ & + \underbrace{((\kappa_{rn,pd}\rho_x - 1) A_{n,x} + \rho_{dn,x})}_{r_{n,x}} x_t + \underbrace{(\kappa_{rn,pd}\rho_{Gn} - 1) A_{n,Gn}}_{r_{n,Gn}} G_{n,t} \\ & + \underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}} \varepsilon_{c,t+1} + \underbrace{(\kappa_{rn,pd} A_{n,G}\sigma_G + \kappa_{rn,pd} A_{n,Gn}\sigma_{Gn,G} + \sigma_{dn,G})}_{\sigma_{rn,G}} \varepsilon_{G,t+1} \\ & + \underbrace{(\kappa_{rn,pd} A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta})}_{\sigma_{rn,\delta}} \varepsilon_{\delta,t+1} + \underbrace{(\kappa_{rn,pd} A_{n,x}\sigma_x + \sigma_{dn,x})}_{\sigma_{rn,x}} \varepsilon_{x,t+1} \\ & + \underbrace{(\kappa_{rn,pd} A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn})}_{\sigma_{rn,Gn}} \varepsilon_{Gn,t+1} + \underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}} \varepsilon_{dM,t+1} + \underbrace{\sigma_{dn}}_{\sigma_{rn,dn}} \varepsilon_{dn,t+1}. \end{aligned} \tag{A.76}$$

We impose the Euler condition

$$\mathrm{E}_t\left[e^{m_{t+1}+r_{n,t+1}}\right] = e^{\kappa_{n,0}+\kappa_{n,Gn}G_{n,t}+\kappa_{n,\delta}\delta_t}, \tag{A.77}$$

where

$$\begin{aligned} m_{t+1} + r_{n,t+1} \simeq\; & m_0 + \kappa_{rn,0} + (\kappa_{rn,pd} - 1) A_{n,0} + \kappa_{rn,pd}(A_{n,G}\mu_G + A_{n,\delta}\mu_\delta + A_{n,Gn}\mu_{Gn}) + \mu_{dn} \\ & + (m_G + (\kappa_{rn,pd}\rho_G - 1) A_{n,G}) G_{W,t} \\ & + (m_\delta + (\kappa_{rn,pd}\rho_\delta - 1) A_{n,\delta} + \rho_{dn,\delta})\delta_t \\ & + (m_x + (\kappa_{rn,pd}\rho_x - 1) A_{n,x} + \rho_{dn,x}) x_t \\ & + (\kappa_{rn,pd}\rho_{Gn} - 1) A_{n,Gn} G_{n,t} \\ & + (-\lambda_c + \sigma_{dn,c})\varepsilon_{c,t+1} \\ & + (-\lambda_G + \kappa_{rn,pd} A_{n,G}\sigma_G + \kappa_{rn,pd} A_{n,Gn}\sigma_{Gn,G} + \sigma_{dn,G})\varepsilon_{G,t+1} \\ & + (-\lambda_\delta + \kappa_{rn,pd} A_{n,\delta}\sigma_\delta + \sigma_{dn,\delta})\varepsilon_{\delta,t+1} \\ & + (-\lambda_x + \kappa_{rn,pd} A_{n,x}\sigma_x + \sigma_{dn,x})\varepsilon_{x,t+1} \\ & + (\kappa_{rn,pd} A_{n,Gn}\sigma_{Gn} + \sigma_{dn,Gn})\varepsilon_{Gn,t+1} \end{aligned}$$

$$+\sigma_{dn,dM}\varepsilon_{dM,t+1}$$
$$+\sigma_{dn}\varepsilon_{dn,t+1}$$
$$=\kappa_{n,0}+\kappa_{n,Gn}G_{n,t}+\kappa_{n,\delta}\delta_t. \quad (A.78)$$

Therefore

$$\begin{aligned}0 =\; & m_0+\kappa_{rn,0}+\left(\kappa_{rn,pd}-1\right)A_{n,0}+\kappa_{rn,pd}\left(A_{n,G}\mu_G+A_{n,\delta}\mu_\delta+A_{n,Gn}\mu_{Gn}\right)+\mu_{dn}\\ & -\kappa_{n,0}+\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}+\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2}\\ & +\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}+\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2}\\ & +\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}+\frac{\sigma_{dn,dM}^2}{2}+\frac{\sigma_{dn}^2}{2}\\ & +\left(m_G+\left(\kappa_{rn,pd}\rho_G-1\right)A_{n,G}\right)G_{W,t}\\ & +\left(m_\delta+\left(\kappa_{rn,pd}\rho_\delta-1\right)A_{n,\delta}+\rho_{dn,\delta}-\kappa_{n,\delta}\right)\delta_t\\ & +\left(m_x+\left(\kappa_{rn,pd}\rho_x-1\right)A_{n,x}+\rho_{dn,x}\right)x_t\\ & +\left(\left(\kappa_{rn,pd}\rho_{Gn}-1\right)A_{n,Gn}-\kappa_{n,Gn}\right)G_{n,t}. \end{aligned} \quad (A.79)$$

Finally, the coefficients in (A.75) are

$$A_{n,0}=\frac{1}{1-\kappa_{rn,pd}}\left(\begin{array}{l} m_0+\kappa_{rn,0}+\kappa_{rn,pd}\left(A_{n,G}\mu_G+A_{n,\delta}\mu_\delta+A_{n,Gn}\mu_{Gn}\right)\\ +\mu_{dn}-\kappa_{n,0}\\ +\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}+\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2}\\ +\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}+\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2}\\ +\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}+\frac{\sigma_{dn,dM}^2}{2}+\frac{\sigma_{dn}^2}{2}\end{array}\right), \quad (A.80)$$

$$A_{n,G}=\frac{m_G}{1-\kappa_{rn,pd}\rho_G}, \quad (A.81)$$

$$A_{n,\delta}=\frac{m_\delta+\rho_{dn,\delta}-\kappa_{n,\delta}}{1-\kappa_{rn,pd}\rho_\delta}, \quad (A.82)$$

$$A_{n,x}=\frac{m_x+\rho_{dn,x}}{1-\kappa_{rn,pd}\rho_x}, \quad (A.83)$$

$$A_{n,Gn}=\frac{-\kappa_{n,Gn}}{1-\kappa_{rn,pd}\rho_{Gn}}. \quad (A.84)$$

Note that $A_{n,Gn}>0$ and that the return coefficient on $G_{n,t}$ is $r_{n,Gn}=\kappa_{n,Gn}<0$. Furthermore, $A_{n,G}>0$, and thus $r_{n,G}<0$, when $\psi>1$. Finally, $A_{n,\delta}$ is positive when

$\bar{G}_n > -\frac{m_\delta}{1-\left(m_\delta+\rho_{dn,\delta}\right)\bar{\delta}}$. We can also rewrite the return on an asset as follows

$$r_{n,t+1} \simeq \underbrace{\left(\begin{array}{l} -m_0+\kappa_{n,0} \\ -\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}-\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2} \\ -\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}-\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2} \\ -\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}-\frac{\sigma_{dn,dM}^2}{2}-\frac{\sigma_{dn}^2}{2} \end{array}\right)}_{r_{n,0}}$$

$$\underbrace{-m_G}_{r_{n,G}}G_{W,t}+\underbrace{\left(\kappa_{n,\delta}-m_\delta\right)}_{r_{n,\delta}}\delta_t\underbrace{-m_x}_{r_{n,x}}x_t+\underbrace{\kappa_{n,Gn}}_{r_{n,Gn}}G_{n,t}$$

$$+\underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}}\varepsilon_{c,t+1}+\underbrace{\left(\kappa_{rn,pd}A_{n,G}\sigma_G+\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn,G}+\sigma_{dn,G}\right)}_{\sigma_{rn,G}}\varepsilon_{G,t+1}$$

$$+\underbrace{\left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)}_{\sigma_{rn,\delta}}\varepsilon_{\delta,t+1}+\underbrace{\left(\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)}_{\sigma_{rn,x}}\varepsilon_{x,t+1}$$

$$+\underbrace{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)}_{\sigma_{rn,Gn}}\varepsilon_{Gn,t+1}+\underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}}\varepsilon_{dM,t+1}+\underbrace{\sigma_{dn}}_{\sigma_{rn,dn}}\varepsilon_{dn,t+1}. \tag{A.85}$$

Proposition 4 is obtained imposing $\rho_{dn,\delta} = \sigma_{dn,G} = \sigma_{dn,\delta} = \sigma_{dn,x} = \sigma_{dn,Gn} = 0$. Recalling (15), the excess return $\hat{r}_{n,t+1} = r_{n,t+1} - r_{f,t+1}$ can be expressed as

$$\hat{r}_{n,t+1} \simeq \underbrace{\left(\begin{array}{l} \kappa_{n,0}-\frac{\left(-\lambda_c+\sigma_{dn,c}\right)^2}{2}-\frac{\left(-\lambda_G+\kappa_{rn,pd}\left(A_{n,G}\sigma_G+A_{n,Gn}\sigma_{Gn,G}\right)+\sigma_{dn,G}\right)^2}{2} \\ -\frac{\left(-\lambda_\delta+\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)^2}{2}-\frac{\left(-\lambda_x+\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)^2}{2} \\ -\frac{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)^2}{2}-\frac{\sigma_{dn,dM}^2}{2}-\frac{\sigma_{dn}^2}{2}+\frac{\lambda_c^2}{2}+\frac{\lambda_G^2}{2}+\frac{\lambda_\delta^2}{2}+\frac{\lambda_x^2}{2} \end{array}\right)}_{\hat{r}_{n,0}}$$

$$+\underbrace{\kappa_{n,\delta}}_{\hat{r}_{n,\delta}}\delta_t+\underbrace{\kappa_{n,Gn}}_{\hat{r}_{n,Gn}}G_{n,t}$$

$$+\underbrace{\sigma_{dn,c}}_{\sigma_{rn,c}}\varepsilon_{c,t+1}+\underbrace{\left(\kappa_{rn,pd}A_{n,G}\sigma_G+\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn,G}+\sigma_{dn,G}\right)}_{\sigma_{rn,G}}\varepsilon_{G,t+1}$$

$$+\underbrace{\left(\kappa_{rn,pd}A_{n,\delta}\sigma_\delta+\sigma_{dn,\delta}\right)}_{\sigma_{rn,\delta}}\varepsilon_{\delta,t+1}+\underbrace{\left(\kappa_{rn,pd}A_{n,x}\sigma_x+\sigma_{dn,x}\right)}_{\sigma_{rn,x}}\varepsilon_{x,t+1}$$

$$+\underbrace{\left(\kappa_{rn,pd}A_{n,Gn}\sigma_{Gn}+\sigma_{dn,Gn}\right)}_{\sigma_{rn,Gn}}\varepsilon_{Gn,t+1}+\underbrace{\sigma_{dn,dM}}_{\sigma_{rn,dM}}\varepsilon_{dM,t+1}+\underbrace{\sigma_{dn}}_{\sigma_{rn,dn}}\varepsilon_{dn,t+1}. \tag{A.86}$$

The expected excess return can also be written as

$$\mathrm{E}_t\left[r_{n,t+1}-r_{f,t+1}\right]+\frac{1}{2}\mathrm{Var}_t\left[r_{n,t+1}\right]=-\mathrm{Cov}_t\left[m_{t+1},r_{n,t+1}\right]-y_{n,t}, \tag{A.87}$$

where

$$
-\mathrm{Cov}_t\left[m_{t+1}, r_{n,t+1}\right] = -\mathrm{Cov}_t\left[\begin{array}{l} -\lambda_c\varepsilon_{c,t+1} - \lambda_G\varepsilon_{G,t+1} - \lambda_\delta\varepsilon_{\delta,t+1} - \lambda_x\varepsilon_{x,t+1}, \\ \sigma_{rn,c}\varepsilon_{c,t+1} + \sigma_{rn,G}\varepsilon_{G,t+1} + \sigma_{rn,\delta}\varepsilon_{\delta,t+1} + \sigma_{rn,x}\varepsilon_{x,t+1} \\ +\sigma_{rn,Gn}\varepsilon_{gn,t+1} + \sigma_{rn,dM}\varepsilon_{dM,t+1} + \sigma_{rn,dn}\varepsilon_{dn,t+1} \end{array}\right]
$$

$$
= \lambda_c \underbrace{\frac{\sigma_{rn,c}}{\sigma_c^2}}_{\beta_{n,c}} \sigma_c^2 + \lambda_G \underbrace{\frac{\sigma_{rn,G}}{\sigma_G^2}}_{\beta_{n,G}} \sigma_G^2 + \lambda_\delta \underbrace{\frac{\sigma_{rn,\delta}}{\sigma_\delta^2}}_{\beta_{n,\delta}} \sigma_\delta^2 + \lambda_x \underbrace{\frac{\sigma_{rn,x}}{\sigma_x^2}}_{\beta_{n,x}} \sigma_x^2, \tag{A.88}
$$

$$
\frac{1}{2}\mathrm{Var}_t\left[r_{n,t+1}\right] = \frac{\sigma_{rn,c}^2}{2} + \frac{\sigma_{rn,G}^2}{2} + \frac{\sigma_{rn,\delta}^2}{2} + \frac{\sigma_{rn,x}^2}{2} + \frac{\sigma_{rn,Gn}^2}{2} + \frac{\sigma_{rn,dM}^2}{2} + \frac{\sigma_{rn,dn}^2}{2}, \tag{A.89}
$$

and

$$
y_{n,t} = -\log\left(1 - \delta_t G_{n,t}\right) \simeq -\left(\kappa_{n,0} + \kappa_{n,\delta}\delta_t + \kappa_{n,Gn}G_{n,t}\right). \tag{A.90}
$$

F Estimation methodology

To perform the estimation, we use the Kalman filter (Hamilton, 1994) to write a likelihood function that is then numerically maximized relative to the parameter space. We first develop the state space representation, jointly considering the equations representing the dynamics of consumption growth in (7), aggregate ESG supply and demand in (9) and (10), long-run risk in (8), the greenness of portfolio j ($j = \{br, neu, gr\}$) in (21), market excess return in (A.69), and individual portfolio excess returns in (A.86):

$$
\Delta c_{t+1} = \mu_c + x_t + \sigma_c\varepsilon_{c,t+1}, \tag{A.91}
$$

$$
G_{W,t+1} = \mu_G + \rho_G G_{W,t} + \sigma_G\varepsilon_{G,t+1}, \tag{A.92}
$$

$$
\delta_{t+1} = \mu_\delta + \rho_\delta\delta_t + \sigma_\delta\varepsilon_{\delta,t+1}, \tag{A.93}
$$

$$
x_{t+1} = \rho_x x_t + \sigma_x\varepsilon_{x,t+1}, \tag{A.94}
$$

$$
G_{j,t+1} = \mu_{Gj} + \rho_{Gj}G_{j,t} + \sigma_{Gj,G}\varepsilon_{G,t+1} + \sigma_{Gj}\varepsilon_{Gj,t+1}, \tag{A.95}
$$

$$
\begin{aligned} \hat{r}_{M,t+1} \simeq\ & \hat{r}_{M,0} + \hat{r}_{M,G}G_{W,t} + \hat{r}_{M,\delta}\delta_t + \hat{r}_{M,x}x_t \\ & + \sigma_{rM,c}\varepsilon_{c,t+1} + \sigma_{rM,G}\varepsilon_{G,t+1} + \sigma_{rM,\delta}\varepsilon_{\delta,t+1} \\ & + \sigma_{rM,x}\varepsilon_{x,t+1} + \sigma_{rM,dM}\varepsilon_{dM,t+1}, \end{aligned} \tag{A.96}
$$

$$
\begin{aligned} \hat{r}_{j,t+1} \simeq\ & \hat{r}_{j,0} + \hat{r}_{j,G}G_{W,t} + \hat{r}_{j,\delta}\delta_t + \hat{r}_{j,x}x_t + \hat{r}_{j,Gj}G_{j,t} \\ & + \sigma_{rj,c}\varepsilon_{c,t+1} + \sigma_{rj,G}\varepsilon_{G,t+1} + \sigma_{rj,\delta}\varepsilon_{\delta,t+1} + \sigma_{rj,x}\varepsilon_{x,t+1} \\ & + \sigma_{rj,Gj}\varepsilon_{Gj,t+1} + \sigma_{rj,dM}\varepsilon_{dM,t+1} + \sigma_{rj,dj}\varepsilon_{dj,t+1}. \end{aligned} \tag{A.97}
$$

Note that the right-hand side depends on the current value of the state variables $G_{W,t}$, δ_t, x_t, and $G_{j,t}$, as well as on the innovations $\varepsilon_{c,t+1}$, $\varepsilon_{G,t+1}$, $\varepsilon_{\delta,t+1}$, $\varepsilon_{x,t+1}$, $\varepsilon_{Gj,t+1}$, $\varepsilon_{dM,t+1}$, and $\varepsilon_{dj,t+1}$. The equations can be stacked through a VAR representation:

$$\boldsymbol{X}_{t+1} = \boldsymbol{A}_{\boldsymbol{X}} + \boldsymbol{B}_{\boldsymbol{X}} \boldsymbol{X}_t + \boldsymbol{\sigma}_{\boldsymbol{X}} \boldsymbol{\varepsilon}_{t+1}, \tag{A.98}$$

where:

$$\boldsymbol{X}_t = \begin{bmatrix} \Delta c_t \\ G_{W,t} \\ \delta_t \\ x_t \\ \vdots \\ G_{j,t} \\ \vdots \\ \hat{r}_{M,t+1} \\ \vdots \\ \hat{r}_{j,t+1} \\ \vdots \end{bmatrix}, \quad \boldsymbol{A}_{\boldsymbol{X}} = \begin{bmatrix} \mu_c \\ \mu_G \\ \mu_\delta \\ 0 \\ \vdots \\ \mu_{Gj} \\ \vdots \\ \hat{r}_{M,0} \\ \vdots \\ \hat{r}_{j,0} \\ \vdots \end{bmatrix}, \quad \boldsymbol{\varepsilon}_{t+1} = \begin{bmatrix} \varepsilon_{c,t+1} \\ \varepsilon_{G,t+1} \\ \varepsilon_{\delta,t+1} \\ \varepsilon_{x,t+1} \\ \vdots \\ \varepsilon_{Gj,t+1} \\ \vdots \\ \varepsilon_{dM,t+1} \\ \vdots \\ \varepsilon_{dj,t+1} \\ \vdots \end{bmatrix}, \tag{A.99}$$

$$\boldsymbol{B}_{\boldsymbol{X}} = \begin{bmatrix} 0 & 0 & 0 & 1 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \rho_G & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \rho_\delta & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \rho_x & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & 0 & 0 & \rho_{Gj} & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \hat{r}_{M,G} & \hat{r}_{M,\delta} & \hat{r}_{M,x} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \hat{r}_{j,G} & \hat{r}_{j,\delta} & \hat{r}_{j,x} & 0 & \hat{r}_{j,Gj} & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & 0 & 0 & \cdots & 0 & \cdots \end{bmatrix}, \tag{A.100}$$

$$\boldsymbol{\sigma_X} = \begin{bmatrix} \sigma_c & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & \sigma_G & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & \sigma_\delta & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ 0 & 0 & 0 & \sigma_x & 0 & 0 & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & \vdots & \cdots & 0 & \cdots \\ 0 & \sigma_{Gj,G} & 0 & 0 & 0 & \sigma_{Gj} & 0 & 0 & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & \cdots & 0 & \cdots \\ \sigma_{rM,c} & \sigma_{rM,G} & \sigma_{rM,\delta} & \sigma_{rM,x} & \cdots & 0 & \cdots & \sigma_{rM,dM} & \cdots & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 & \vdots & \ddots & 0 & 0 \\ \sigma_{rj,c} & \sigma_{rj,G} & \sigma_{rj,\delta} & \sigma_{rj,x} & 0 & \sigma_{rj,Gj} & 0 & \sigma_{rj,dM} & 0 & \sigma_{rj,dj} & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 & 0 & \ddots & \vdots & 0 & 0 & \ddots \end{bmatrix}. \tag{A.101}$$

We consider as observables the real monthly consumption growth, the ESG scores of the market (proxying for the greenness of the aggregate wealth portfolio) and its excess return, as well as the ESG scores of the portfolios and their monthly returns. We stack these variables in the vector $\boldsymbol{Y}_t$:

$$\boldsymbol{Y}_t = \begin{bmatrix} \Delta c_t & G_{W,t} & \cdots & G_{j,t} & \cdots & \hat{r}_{M,t} & \cdots & \hat{r}_{j,t} & \cdots \end{bmatrix}'. \tag{A.102}$$

The observation equation of the Kalman filter (with zero observation errors) is given by

$$\boldsymbol{Y}_t = \boldsymbol{H}\boldsymbol{X}_t, \tag{A.103}$$

and $\boldsymbol{H}$ is a sparse matrix loading with unit weights the elements of $\boldsymbol{X}_t$ that belong to $\boldsymbol{Y}_t$:

$$\boldsymbol{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{A.104}$$

The prediction stage is described by the following transition equations, which provide the time-t conditional expectation and covariances of the state variables in $t+1$:

$$\boldsymbol{X}_{t+1|t} = \boldsymbol{A_X} + \boldsymbol{B_X}\boldsymbol{X}_{t|t}, \tag{A.105}$$

$$\boldsymbol{\Sigma}^X_{t+1|t} = \boldsymbol{B_X}\boldsymbol{\Sigma}^X_{t|t}\boldsymbol{B}'_{\boldsymbol{X}} + \boldsymbol{\sigma_X}\boldsymbol{\sigma}'_{\boldsymbol{X}}, \tag{A.106}$$

$\boldsymbol{X}_{1|0}$ is initialized considering the initial values of the observable variables, complemented by δ_0 and x_0, which belong to the parameter space and represent the unobservable initial values of the processes δ_t and x_t. $\boldsymbol{\Sigma}^X_{1|0}$ is initialized at $\boldsymbol{\sigma_X \sigma'_X}$. The predicted vector of observables is thus $\boldsymbol{Y}_{t+1|t} = \boldsymbol{H}\boldsymbol{X}_{t+1|t}$. The updating equations, which consider the $t+1$ observed values $\boldsymbol{Y}_{t+1}$, are then

$$\boldsymbol{X}_{t+1|t+1} = \boldsymbol{X}_{t+1|t} + \boldsymbol{K}_{t+1}\left(\boldsymbol{Y}_{t+1} - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right), \tag{A.107}$$

$$\boldsymbol{\Sigma}^X_{t+1|t+1} = \boldsymbol{\Sigma}^X_{t+1|t} - \boldsymbol{K}_{t+1}\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)\boldsymbol{K}'_{t+1}, \tag{A.108}$$

where $\boldsymbol{K}_{t+1} = \boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)^{-1}$ is the Kalman gain. Given a candidate set of model parameters Θ, equations (A.105) through (A.108) are evaluated recursively. Then, for each time step, the following log-likelihood function is evaluated

$$\begin{aligned}\ell_{t+1}(\Theta) = &-\frac{n}{2}\log(2\pi) - \frac{1}{2}\log\left|\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right| \\ &-\frac{1}{2}\left(\boldsymbol{Y}_t - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right)'\left(\boldsymbol{H}\boldsymbol{\Sigma}^X_{t+1|t}\boldsymbol{H}'\right)^{-1}\left(\boldsymbol{Y}_t - \boldsymbol{H}\boldsymbol{X}_{t+1|t}\right).\end{aligned} \tag{A.109}$$

The total log-likelihood, $\ell(\Theta) = \sum_{t=1}^{T}\ell_t(\Theta)$, is numerically maximized with respect to the parameter space Θ to obtain the model estimates. In the optimization, we impose the long-run means of the aggregate ESG score, $\bar{G}_W$, and the individual asset scores, $\bar{G}_n$, to be equal to their sample means. Similarly, $\bar{\delta}$ is equal the sample mean of the filtered state variable δ_t. We further set the long-run means of the model-implied price-to-dividend ratios of the market portfolios and individual assets to match the sample average of the observed price-to-dividend ratios. Finally, δ_t is restricted to be nonnegative.

G Shocks to ESG demand and ESG score in the presence of correlated casflows

We perform a supplementary analysis where dividend growth is allowed to be correlated with innovations of ESG demand and of the asset's ESG score. This implies relaxing the hypothesis that the coefficients $\sigma_{dn,\delta}$ and $\sigma_{dn,Gn}$, appearing in equation (A.74), are equal to zero.

To allow for a conditional correlation between dividend growth and ESG demand, $\text{Corr}_t\left[\Delta d_{n,t+1}, \delta_{t+1}\right]$, we consider the baseline parameter values reported in Section 4.2 and replace $\sigma_{dn,\delta}$ (which baseline value is zero) and σ_{dn} with $\tilde{\sigma}_{dn,\delta}$ and $\tilde{\sigma}_{dn}$, respectively, such that i) the conditional correlation between dividend growth and ESG demand equals the value we aim to impose, and ii) the total dividend growth volatility, $\sigma_{dn,tot}$, is the same as the estimated one:

$$\sigma_{dn,tot} = \sqrt{\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM} + \sigma^2_{dn}}, \tag{A.110a}$$

$$\tilde{\sigma}_{dn,\delta} = \sigma_{dn,tot} \cdot \text{Corr}_t \left[\Delta d_{n,t+1}, \delta_{t+1}\right], \tag{A.110b}$$

$$\tilde{\sigma}_{dn} = \sqrt{\sigma^2_{dn,tot} - \left(\sigma^2_{dn,c} + \sigma^2_{dn,G} + \tilde{\sigma}^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM}\right)}. \tag{A.110c}$$

Similarly, to allow for a conditional correlation between dividend growth and the asset's ESG score, $\text{Corr}_t \left[\Delta d_{n,t+1}, G_{n,t+1}\right]$, we determine $\tilde{\sigma}_{dn,Gn}$ and $\tilde{\sigma}_{dn}$ such that:

$$\sigma_{dn,tot} = \sqrt{\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \sigma^2_{dn,Gn} + \sigma^2_{dn,dM} + \sigma^2_{dn}}, \tag{A.111a}$$

$$\tilde{\sigma}_{dn,Gn} = \sigma_{dn,tot} \cdot \text{Corr}_t \left[\Delta d_{n,t+1}, G_{n,t+1}\right], \tag{A.111b}$$

$$\tilde{\sigma}_{dn} = \sqrt{\sigma^2_{dn,tot} - \left(\sigma^2_{dn,c} + \sigma^2_{dn,G} + \sigma^2_{dn,\delta} + \sigma^2_{dn,x} + \tilde{\sigma}^2_{dn,Gn} + \sigma^2_{dn,dM}\right)}. \tag{A.111c}$$

The graphs on the left in Figure A.1 show the response to an unexpected one-standard deviation annual shock of ESG demand for different values of the correlations $\text{Corr}_t \left[\Delta d_{gr,t+1}, \delta_{t+1}\right]$ and $\text{Corr}_t \left[\Delta d_{br,t+1}, \delta_{t+1}\right]$. The graphs on the right show the response to an unexpected +0.1 annual shock of the green asset's ESG score, $G_{gr,t+1}$, for different values of the correlation $\text{Corr}_t \left[\Delta d_{gr,t+1}, G_{gr,t+1}\right]$.

Figure A.1: Impact of casflows' correlation with ESG demand and ESG score.

The graphs on the left show responses to a one-standard deviation positive annual shock applied to δ_t. Solid lines correspond to zero correlations, $\text{Corr}_t\left[\Delta d_{gr,t+1}, \delta_{t+1}\right]$ and $\text{Corr}_t\left[\Delta d_{br,t+1}, \delta_{t+1}\right]$, between portfolio dividend growth and δ_t. Dashed lines correspond to a negative (positive) correlation for the green (brown) asset. Dotted lines correspond to a positive (negative) correlation for the green (brown) asset. The graphs on the right show responses to a +0.1 annual shock to the ESG score of the green portfolio. Solid lines correspond to a zero correlation $\text{Corr}_t\left[\Delta d_{gr,t+1}, G_{gr,t+1}\right]$, dashed (dotted) lines to a negative (positive) correlation. The expected and realized excess returns of the brown and green portfolios, the cumulative return of the green-minus-brown portfolio, and the price-to-annual dividend ratios of the brown and green portfolios are shown. The state variables are initially set at their average values and the shocks are equally distributed throughout 12 consecutive months.